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Return on Investment on Naval Education & Research

Mun, Johnathan

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Naval Research Program

Return on Investment on Naval Education & Research

Dr. Johnathan Mun

Research Professor

Naval Postgraduate School

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ABSTRACT

The United States Navy and other military services send a large number of their officers to various military universities to obtain graduate degrees or perform academic research. These graduate education programs provide the officers with technical skills and nontechnical competencies highly valued in their respective billets. The cost of sending a Navy officer to a 1.5- to 2-year program for a master's degree may be upwards of $250,000 plus the opportunity cost of his or her lost services. In addition, a doctoral program may cost upwards of $500,000 per officer, plus their respective soft opportunity costs for being away for 3–4 years. The U.S. military's human resource environment is unique in that it is a closed internal hierarchical structure. For instance, an officer's pay is based on his or her rank and years of service, regardless of educational background. It can be argued that higher education may result in higher efficiency and productivity, thereby increasing the speed of promotions, but these are fairly difficult to quantify. Further, we see that 2 years after graduation, the retention rates are relatively high, ranging from 99.31% to 95.78% on average. This high rate of retention the first few years is to be expected as officers sent to graduate programs typically are required to "pay back" their education costs with guaranteed service for several years. The question is whether the benefits of such education and research are indeed greater than the cost incurred by the Navy. Another consideration is that naval research and education are not separate tasks but tend to coexist alongside the innovation engines of the country.

The current research looks at various novel ways to value the monetary return on investment (ROI) of military education and research. The proposed methodologies apply theoretical constructs by using a systems approach to utilization; convolution methods to determine the frequency and quantity of use; and an analytical framework, empirical impact analysis, and work lifecycle approach, combined with integrated risk management and knowledge value added methodologies to determine and run Monte Carlo simulations of the model inputs, as well as to provide guidance and information to decision makers with respect to the optimal portfolio allocation of resources to educational activities.

The research also includes an examination of three short case studies: one on the value of military research in the Naval Acquisitions Research Program, a second case study on the value of a naval university such as the Naval Postgraduate School (NPS) in Monterey, California, and a third case on the Defense Acquisition University. The research findings indicate that there is a statistically

significant positive impact on retention of graduating officers, lower attendance cost, and greater DoD control of the courses covered. In fact, the ROI for military-based academic research ranges between **240%** and **600%**, while graduate education at a military university such as NPS yields an ROI between **469%** and **673%**. The courses at the DAU have an average ROI between **411%** and **477%**, and the probability that, on average, any given course taken at the DAU has at least a **93%** probability that the ROI is positive for an organization. The global average ROI for various military education is estimated to be about **485%**, which means that for every **$1** spent on education, the benefit gained by the government is **$5.85**. These ROI are above and beyond the significant intangible value of military officers studying a military-specific curriculum and learning from each other as well as from retired military faculty. Finally, we also conclude that military organizations tend to value the ROI to an employee's personal career growth as being the same as the ROI to the entire organization, where the ROI of a training initiative goes well beyond its sole impact on an employee's job performance where the value add might be intrinsic, unmeasurable, and subjective, rather than simple applications of specific knowledge or learned skill set on the job.

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I. INTRODUCTION

According to the Department of the Navy's "Education for Seapower (E4S) Report" (2018), "continuous learning and sharing hard-won knowledge represents a combat-proven key to victory for our naval services." The U.S. Department of Navy (DON) flagship educational institutions include the United States Naval Academy (USNA), Naval Postgraduate School (NPS), Marine Corps University (MCU), and Naval War College (NWC), as well as other outstanding national colleges and universities associated with the Reserve Officers Training Corps (ROTC), that have long and well served the country in educating our future military and civilian leaders. These institutions "inculcate not only the finest sense of honor and integrity, but also creativity and deep rigor in thinking about the future of naval warfare, especially in times of great change" (Department of the Navy, 2018).

The DON and other military services send a large number of their mid-level officers (mostly O-3 and O-4 levels) to graduate programs to obtain advanced degrees. These graduate education programs provide the officers with technical skills and nontechnical competencies highly valued in their respective billets. As an example, the cost of sending a Navy officer to a 1.5- to 2-year program for a master's degree may cost upwards of $250,000 plus the opportunity cost of his or her lost services. In addition, a doctoral program may cost upwards of $500,000 per officer, plus their respective soft opportunity costs for being away for 3–4 years. The question is whether the benefits of such education are indeed greater than the cost incurred by the DON. The current research looks at various novel ways to value the monetary return on investment (ROI) of these military education and research activities.

The value of education and research has always been a simple concept to understand but a fairly difficult one to measure. For instance, one can generally agree that higher education has value to the individual, both in terms of economic returns as well as incalculable and intangible values such as the deepening of one's knowledge perspective and enrichment of one's experience of the world. "The U.S. Navy invests over $3.3B across the FYDP at NPS, NWC and civilian schools" (Department of the Navy, 2018), and in the past, the ROI in sending officers to such in-residence on-campus education programs has been measured, to some degree, by retention or years of service beyond the education. The assumption is that these officers will apply the knowledge and skills learned in their respective billets or positions. Retaining our warfighting top talent and broadening their skill sets with the strategic and critical thinking attributes honed by these educational programs help build an officer corps that would be more capable at executing the DON's maritime strategy.

According to the Department of the Navy (2018):

> Education has long been the key strength of the American naval profession and a force multiplier for our Sea Services. Changes in society, technology, and our security environment are occurring at a rapid pace. Failure to adapt all aspects of how we prepare our naval leaders for the future creates unacceptable risk for American citizens, who have long relied on the Naval Services to be at the intellectual forefront of national security concerns. In order to ensure that our Navy and Marine Corps are prepared for the complexity and rapidity of the modern world, we must educate leaders who have the skills required to solve problems that cannot even be imagined today.

The E4S report continues by stressing that, from sand table exercises at the most junior level, to complex war-games simulating theater, cyber, digital, global, or space conflict, the capacity of mindful decision may be one of the most strategically important outcomes of the education of a naval leader. The report continues to highlight that history is replete with examples of leaders at all levels who were immobilized at the moment of truth because they neither possessed the base knowledge to decide, nor did they possess the capacity to decide and act. Our future will similarly demand leaders who possess both the knowledge accumulated from all the elements of naval education previously discussed in this vision statement, as well as the moral capacity to decide and act (Department of the Navy, 2018).

The proposed methodologies in this current research apply theoretical constructs by using a systems approach to utilization of knowledge gained, mathematical convolution methods to determine the frequency and quantity of use to determine the expected returns, an analytical framework to apply econometric models, empirical impact assessments, and simulated work lifecycle approach of the individual combined with integrated risk management and knowledge value added methodologies. The discussion of each method will include the underlying theory as well as stylized examples of their applications.

Research Motivation

In February 2019, the DON issued its critical and landmark report, “Education for Seapower (E4S) Report,” which focused on recommending major reforms and improvement of the current naval education for commissioned and enlisted personnel in the Navy and Marines. At the direction of Secretary of the Navy Richard V. Spencer, through his memorandum to all naval forces, the Department of the Navy has started to implement the report’s recommendations. According to John Kroger, the Chief Learning Officer of the Department of the Navy, there were 10 main takeaways from the detailed “Education for Seapower Report” (Kroger, 2019):

- Education of our force is vital to national security.
- Our current educational efforts are inadequate.
- Immediate action is necessary.
- We must invest in and support our educational institutions.
- We must create a Naval Community College for enlisted personnel.
- We need 21st-century education.
- We must adopt school selection standards.
- The Navy must change its evaluation and promotion system to value education.
- Leaders must take responsibility for education in their command.
- Improving education is a team effort.

The DON is a diverse and deployable force, which means that experience at sea has always been more valued over formal education, meaning that the perspective is such that the ROI on formal education would not be as high as spending resources to increase basic seamanship and military proficiency. Indeed, the current system seems to not affirm that there is a return on investment in education. “A word count of the interview content labeled under *Problem and Culture* has shown that the DON’s education system has a deeply ingrained culture that does not view education in high regard. The system places more importance on experience and career and does not justify the value education can bring to one’s career” (Department of the Navy, 2018). One can infer, then, that the current system discourages personnel from furthering their education. Education seems to only develop a “habit of mind, that does not immediately evidence itself in all cases. As a result, the return on investment is hard to recognize in the near term – in essence, educators are venture capitalists" (Department of the Navy, 2018).

A RAND 2010 research indicated that, "the overall benefits in terms of ROI to the Navy from graduate education can be measured, given certain assumptions" (Kamarck, Thie, Adelson, & Krull, 2010). But the report continues with a highly simplistic set of assumptions to generate said ROI. For instance, the very detailed report spends most of its 110 pages analyzing the political landscape, military policies, and guidance on education, but includes only one paragraph explaining the potential value benefits of an officer with a graduate degree, specifically, making highly dubious, generalized, and subjective rough order magnitude estimates that there will be a "20% productivity gain and 5% skill productivity differential of an officer with graduate education than one without" and that "ROI can only be justified with an officer's long continued service and reutilization post-education" (Kamarck et al., 2010, pp. xvii–xviii, 49–50). This indicates that even a detailed study performed by one of the world's most prestigious think tanks falls short of determining an adequately robust ROI measure for military education.

The RAND research only reinforces the fact that ROI determination in military education is not an easy undertaking. Therefore, this current research will not evaluate the efficacy of the political status or policy deliberations but will focus on a singular goal: determining a set of potentially viable methodologies and techniques from which a robust ROI for military education and research can be determined. Computing the actual ROI requires a longer research project where the collection of actual data from current and former graduate students, their current billets and performance, will be required, and hence, falls outside the scope of this current research.

Research Objective and Problem Statement

The Navy's investment in education must be "fiscally disciplined focusing on the tenants of Warfighting First, Operate Forward, and Be Ready" (Department of the Navy, 2018). There is a need to align education resources with the highest priorities and return on investment. The current research examines the challenges of determining the ROI of military education. The primary objective of the research is to provide a set of recommendations and methodologies, as well as additional insights and examples of how some of these methods can be applied.

Research Questions

The questions examined in this research are as follows:

1. How can ROI be defined and calculated within the realms of military education?
2. What is the ROI of military education and research?
3. How can we determine the optimal allocation of resources and investments among competing initiatives in the DON, from multidomain activities to education for its officers?

Technical Approaches and Outcomes of the Research

Various technical approaches are proposed in this research to extract the valuation of an ROI for military education and research. There are three main areas: (i) theoretical constructs, where various underlying theories in economics, finance, mathematics, and decision sciences are brought to bear; (ii) integrated risk management, where advanced Monte Carlo simulation of the lifecycle of value-added benefits of education are run, and portfolio optimizations are executed to determine the ROI and benefit of military education; and (iii) knowledge value added, where intangible and non-economic values can be monetized to generate quantifiable values to determine educational ROI. All three groups of methods are utilized in the case study presented in section five of this research.

As explained previously, this research dispenses with the discussions of the softer side of the benefits of graduate education, which, in most cases, we can all agree are invaluable. For instance, we will not delve into the area of social capital theory, psychosocial emotional theory, or human capital theory. Clearly, higher education, when done properly, will enhance one's soft skill competencies (good judgement, better perception, risk management skills, common sense, presentation skills, leadership skills, etc.), but these are very difficult to quantify and convert to a numerical ROI. Therefore, this current research focuses on more tangible skills that can be valued and modeled into an ROI measure.

Theoretical Constructs

Various theoretical approaches are examined in this research, from the systems approach with utilization metrics, frequency and quantity of use, and analytical framework approach, to an empirical impact and work lifecycle approach. These methods will be combined with the modern data science and decision analytics approaches, such as integrated risk management and knowledge value added, to triangulate the ROI of military education.

Integrated Risk Management

IRM is a comprehensive methodology that is a forward-looking risk-based decision support system incorporating various methods such as Monte Carlo Risk Simulation, Stochastic Forecasting, Portfolio Optimization, Strategic Flexibility Options, and Economic Business Case Modeling. Economic business cases using standard financial cash flows and cost estimates, as well as non-economic variables such as expected military value, strategic value, and other domain-specific subject matter expert (SME) metrics (e.g., Innovation Index, Conversion Capability, Ability to Meet Future Threats, Force Structure, Modernization and Technical Sophistication, Combat Readiness, Sustainability, Future Readiness to Meet Threats) can be incorporated (Mun, 2016). These metrics can be forecasted as well as risk-simulated to account for their uncertainties and modeled to determine their returns to education cost (e.g., return on investment for innovation, or return on sustainability). Capital investment and acquisition decisions within education portfolios can then be tentatively made, subject to any budgetary, billet requirements, and knowledge capability constraints. Portfolio management is often integrated with IRM methods to provide a more holistic view in terms of educational programs.

Knowledge Value Added

KVA identifies the actual cost and value of an organization's assets (human, educational, and technological), standard functional areas, or core processes. KVA identifies every process required to produce an output, and the historical costs of those processes, the unit costs and unit values of products, processes, functions, or services can be measured. By describing processes in common units, the methodology also permits market-comparable data to be generated; this ability is particularly important for nonprofits like the military and government organizations. Value is quantified in two key productivity metrics: Return on Knowledge (ROK) and Return on Knowledge Investment (ROKI). KVA includes the following seven-step method (Housel & Kanevsky, 2006):

- Identify functional areas and core processes along with their subprocesses. It is quite useful to have at least two process or functional area SMEs to ensure reliable estimates.

  - Establish common units and level of aggregation of the process output to measure learning time. Other common-unit measures of output can also be used such as tasks, computer code, or process instructions that may be contained in existing documentation as long as they are calibrated to a common level of complexity using learning times.
  - Calculate learning time (i.e., knowledge surrogate) required to execute each process or functional area.
  - Designate a sampling time period long enough to capture a representative sample of the core processes' or functional area's aggregated output.
  - Multiply learning time for each process by the number of times the process executes during the sample period.
  - Calculate the cost to execute knowledge (e.g., learning time or process instructions) by the resource used to produce the outputs (i.e., people, technology) to determine process costs.
  - Calculate ROK and ROKI.

Research Report Layout

The next section provides a detailed list of the literature survey performed, with an emphasis on ROI in general as well as within the realms of military education and research. The third section delves into the intricacies of the proposed theoretical constructs of valuing ROI in education and research. The fourth section lists some additional enhanced methodologies in more detail, as well as provides examples of how they are applied. The fifth section looks at a sample case study of research in the acquisitions research program at NPS. The sixth section reviews the ROI of the Naval Postgraduate School, and the seventh section looks at the value add of courses held by the Defense Acquisition University. The final section wraps up the research with a series of conclusions and recommendations.

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II. LITERATURE SURVEY

This section starts with a discussion of the challenges to computing ROI in education in general, followed by the challenges faced in military education and research. This review provides only a basic understanding of these methods whereas the next section goes into each methodology in more detail.

In general, businesses have to question the value of their training and educational investments, as well as balance them against other investment opportunities that are more cut-and-dried. For instance, invest in a certain machine and it generates a higher production output that can be measured and, in turn, generate additional revenue against the original investment. In such situations, ROI on the machine can be computed without much consternation. However, when evaluating the value add of education, the math becomes more complicated if not intractable. Companies that operate under the assumption that positive benefits result from their training efforts might ask their human resource managers to provide proof that their training programs result in positive ROI. A cost-benefit analysis similar to the one used in the machine problem is one means of evaluating training returns because it provides tangible monetary evidence of affecting bottom-line profits.

If the reasons for evaluating training are to ensure a correlation between training and a specific outcome, a detailed level of evaluation may be required. However, "if the reason for training is to improve soft areas such as customer satisfaction, employee morale, and so forth, other methods, such as surveys and interviews, may provide the evidence required to support training" (Brown, 2001), and the literature provides sufficient evidence to support the assumption that investment in training will ultimately result in positive ROI. While the nature of these returns and their impacts may vary among organizations and workers, it is "important to remember that wages and productivity are not the only variables guiding a company's investment in training" (Brown, 2001).

A broader understanding of so-called *soft impact* may help governments and the DON to measure the diverse benefits of their investment in research and education. According to Eisenstein (2016), who performed empirical assessment of outcomes and returns from funding agencies such as the National Institutes of Health (NIH) and National Science Foundation (NSF), "when the Congressional Budget Office does simulations of the effects of investment in areas like tax or education policy, they have models and processes, but when it comes to science, essentially all we say is to send more money." In addition, "the temptation to come up with a number for an impressive-looking economic return can be strong," says Adam Jaffe, director of Motu Economic and Public Policy Research in Wellington, New Zealand, "but I'd argue that you should look at a range of different indicators, including qualitative information" (Eisenstein, 2016). Patents based on academic research may be able to provide a useful indicator of commercial interest in a particular invention. However, not all patents become products and public-sector origins of private-sector patents are not always obvious.

According to Eisenstein (2016), "people tend to use at least 20-year time windows. You can't expect any economic impact in the narrow sense from a research programme within two or three years—that's only the case for exceptional research breakthroughs." He also noted that many independent analyses described a consistent gap of up to 17 years from initial publication of a research finding to economic impact across various biomedical fields.

In another study, Wang, Dou, and Li (2002) explore an interdisciplinary approach for ROI in human resource development (HRD) research and practices, surveying areas of economics, industrial-organizational psychology, financial control, and HRD fields, to develop a systems approach to quantitatively measure ROI for HRD programs. The "applicability of using statistical and mathematical operations to determine ROI and isolate non-HRD program impacts was discussed and application scenarios are presented to demonstrate the utility of the systems approach in real-world ROI measurement for HRD interventions" (Wang et al., 2002).

Challenges in Computing Return on Investment in the Military

A decision maker's primary responsibility is how to decide which investment alternatives provide the greatest return with least risk of loss. In civilian organizations, numerous methods and models assist with these decisions, as will be discussed later in the integrated risk management approach. However, in military and government agencies, these methods often fall short because typical governmental and military investments do not provide for a monetary return. The processes underpinning governmental resource allocation and acquisition decisions are often cumbersome and time consuming. MacLeod and Dinwoodie (2016) present a "unique application of composite indexing methods to compare the return on investment in military equipment." They assume that this method can improve government agencies' investment decisions for capital equipment acquisition, especially when methods that are more laborious cannot be executed in the allotted time frame (MacLeod & Dinwoodie, 2016).

As previously stated, a primary decision-making concern is how to select, among a range of investment alternatives, the option or options that provide the greatest return with least risk of loss. "In civilian organizations, numerous methods and formulas such as net present value, return on investment, and return on assets address these issues" (Brealy, Myers, & Allen, 2011). In military organizations, investments do not offer monetary returns, but they provide "intangible returns such as national defense, public safety, goodwill, and other public goods that are difficult, but not impossible, to quantify" (Oswalt et al., 2011). As Gonzalez, Perera, and Correa (2003) noted, "the economic valuation of nonmarket goods…is aimed at obtaining a monetary assessment of the welfare or utility gain (or loss) experienced by a certain group of people from the improvement of (or damage to) a nonfinancial asset."

Numerous economic models for calculating ROI exist, and most require only a few basic inputs such as "costs, benefits, time horizon, and risks" and that the "benefit of calculating ROI of government investments is to save costs over other alternatives" (Bailey, Mazzuchi, Sarkani, & Rico, 2014), but scholarly research into assessing the ROI of complete military systems is lacking or, at least at the time of writing, insufficient and unsatisfying. In MacLeod and Dinwoodie's (2016) article, they presented a method that efficiently compares equipment options using a "composite index that generates a normalized measure of performance return." By objectively assessing various equipment's ROI, decision makers can eliminate low-value and inefficient programs, ultimately saving U.S. taxpayer dollars.

Determining ROI is fairly straightforward if costs and revenues can be directly identified or attributable to a project, program, or activity. Difficulties arise when indirect costs or returns, as well as soft skills and intangibles exist, which may or may not be associated with the decision and may or may not necessarily be caused by it. The simplest ROI approaches isolate the incremental economic gain from an action by its marginal investment costs. Hence, the higher the ROI, the greater the financial return or bang for the buck for the given investment and the better use of financial resources. "Proximal measures of cost and gains, or returns, can be included, insofar as they can be identified with the specific investment and tracked for sufficient length of time. The analysis grows in complexity with the recognition of several important dimensions of the economic value implied by the ratio, the most important being the timing of the respective cost outlays and revenue inflows" (Grazier, Trochim, Dilts, & Kirk, 2013).

Return on Investment in Military Education

The U.S. Navy and the other military services send a number of their officers to graduate-level institutions each year to obtain advanced degrees primarily to fill positions in their services whose duties require the knowledge and skills gained in graduate school. Furthermore, the benefits of a graduate education extend beyond the specific assignment for which the officer was educated, applying to subsequent assignments as well. The estimated cost of $250,000 to $300,000 per officer for a master's degree is substantial. For fully funded education, the service must pay not only the cost of the education but also the pay and allowances associated with an officer's billet allocated for education as well as assume the opportunity cost of the missing officer's services, and that same officer will also have to forgo any experience that might have been gained while he or she is in school. The question, therefore, is whether the benefit gained from a graduate military education is worth the high cost. "Evaluating the qualitative effects of a graduate education poses a number of challenges. DoD educational policy suggests broader and more extensive use of graduate education than simply filling billets that have been determined to require it" (Kamarck, Thie, Adelson, & Krull, 2010).

Graduate education options include funding for officers who attend as O3 and O4 levels at NPS or other civilian institutions, and, according to Mehay and Bowman (2007), the "Immediate Graduate Education (IGE) program, an alternative that allows qualified newly commissioned ensigns to receive masters' degrees early in their careers. The policy issues surrounding these programs involve analysis of economic ROI on IGE programs, which is needed to guide Navy's strategic human capital decisions."

Return on Investment in Research

University research in the United States is world-class but in order to continue such leadership requires major funding. Public and private sectors have risen to meet that financial need through increased support of university research. "Since 1995, New York Governor George E. Pataki and the New York Legislature have fostered the growth of high technology and biotechnology industries by investing more than $1 billion in superlative research laboratories and academic centers" (Bessette, 2003). However, with this increased investment, there is need for greater accountability. Bessette (2003) recommends that "public funding agencies quantify and tabulate research outputs such that economic impacts are reported as a percent ROI. With this model, multiple stakeholders can evaluate divergent research technologies using a measurement that is familiar to scientists, business leaders, elected officials, and the public."

"The Governor's Office of Indiana requested an annual financial analysis of the INDOT Research Program to determine ROI. The ROI analysis performed supplemented the annual IMPACT report (qualitative and quantitative benefits) by adding a more rigorous quantitative benefit cost analysis (BCA) to the Research Program" (McCullouch, 2018). Previous financial analyses calculating net present values of cash flows to determine a benefit-cost analysis use the same approach.

Holbrook et al. (2009) researched the economic benefits to British Columbia of graduate students trained in research, and the economic and social returns of investment in research. They also looked at knowledge as a commodity, and the conditions of its production. The central question of the 2009 report by Holbrook et al. was "what does a person's ability to seek new knowledge generate as an advantage over human capital based only in established knowledge? In other words, what is the incremental return on investment on research expenditure (IRRE) in the form of human capital generated through publicly funded, paid (not scholarships) research activity?" Determining the ROI in human capital requires a large software system for tracking and understanding individuals' career paths, specific contributions, achievements, and earning power growth over time.

"Public research universities face many challenges in the 21st century, not the least of which involves documenting the value-added outcomes that derive from the teaching, research, and public service missions of the institution. Governing boards, accrediting bodies, funding agencies, state legislators, taxpayers, and the American citizenry in general want to know" (Trewyn, 2001). In fact, investment bankers and stockbrokers should not be the sole individuals interested in ROI; a university's prospective students and parents want to know what sort of ROI can be obtained from the education program. "Universities, just like other entities seeking monetary investments, will be well served if they can provide compelling answers to questions about the ROI they generate in fulfilling their missions" (Trewyn, 2001).

However, estimating the ROI in scientific research proves to be elusive and difficult. According to Grant and Buxton (2018), "you need to be able to value benefits in monetary terms; the time between investment and return is typically long; research is an international endeavor making it difficult to attribute returns to national investments; and you need lots and lots of data over long periods of time." But the problem is, "a massive amount of intellectual capital gets created every day from $150 billion in annual research funding allocated to federal laboratories and universities in the United States. Unfortunately, most of that intellectual capital never makes it to the market and does not generate any ROI" (Nag, 2018).

Oswalt et al. (2011) discuss an approach to comparing different modeling and simulation (M&S) investment opportunities using an ROI-like measure. The authors described methods to evaluate the "benefit" (i.e., increased readiness, more effective training, etc.) received from an investment and use the metrics generated in a decision analysis framework to evaluate each M&S expenditure. They concluded by discussing the importance of viewing M&S investments from a DoD enterprise point of view, evaluating investments over multiple years, measuring well-structured metrics, and using those metrics in a systematic way to produce an ROI-like result that the DoD can use to evaluate and prioritize M&S investments.

III. PROPOSED THEORETICAL CONSTRUCTS IN ROI MODELING

Various theoretical approaches are examined in this section, starting from the *systems approach with utilization metrics*, where the ROI can be determined using production outputs. Next, the *frequency and quantity of use approach* looks at both the frequency and quantity of learned knowledge used in order to determine the value of the knowledge learned. An *analytical framework approach* is used if cross-sectional data can be gathered. The *empirical impact approach* can be used to determine if, indeed, statistically significant value-add exists in post-training compared to situations without any training. Finally, the *work lifecycle approach* can be used to determine the lifecycle valuation of education. These methods will be combined into a singular robust set of methods with modern data science and decision analytics approaches such as *integrated risk management* and *knowledge value added*, as discussed in more detail in Section IV, to simulate and triangulate the ROI of military education.

Systems Approach with Utilization Metrics

The standard utility model originally proposed by Schmidt, Hunter, and Pearlman (1982) can be adapted to a more modern systems approach with the utilization model specified as:

$$\delta U = N[(\Phi_T - \Phi_{UT})\Omega\sigma - C] \qquad \text{(Equation 1)}$$

where δU is the net monetary value of training; N is the number of trained individuals; Φ is the output generated by trained, T, and untrained, UT, individuals; Ω is the duration of the training; C is the cost of the training; and σ is the standard deviation of the performance output of the untrained group. Therefore, $ROI = \frac{\delta U}{C} \times 100\%$.

As an example, suppose we have a group of 10 programmers undergoing a new agile computing training course, which costs the company $35,000 to send the team for this four-month semester-long course. The pre-course and post-course output in terms of *delivered software lines of code* (DSLOC) are collected and a monetary value assigned to each line of code. This was done by looking at the average software delivered by the company and sold in the market. The DSLOC went from 1.1 million per year to 1.6 million per year, per person. The computed ROI is therefore 257%.

Frequency and Quantity of Use Approach

To quantify the value of the knowledge learned, the frequency and quantity of use approach looks at both the frequency and quantity of learned knowledge used. Specifically, let *X*, *Y*, and *Z* be real-valued random variables whereby *X* and *Y* are independently distributed with no correlations. Further, we define F_X, F_Y, and F_Z as their corresponding cumulative distribution functions (CDFs), and f_X, f_Y, f_Z as their corresponding probability density functions (PDFs). Next, we assume that *X* is a random variable denoting the *frequency* that a certain type of learned knowledge is triggered or used and is further assumed to have a discrete Poisson distribution. *Y* is a random variable denoting the *quantity* or amount of the learned knowledge that is used (this can be converted into monetary value or some other economic value or kept simply as an index of output or output ratios such as those computed using the Knowledge Value Added methodology shown in Section IV) and can be distributed from among a group of continuous distributions (e.g., Fréchet, Gamma, Log Logistic, Lognormal, Pareto, Weibull, etc.).

Therefore, *Frequency* × *Quantity* equals the *Total Unit Quantified*, which we define as Z, where $Z = X \times Y$ (Mun, 2016).

Then the Total Usage formula yields:

$$F_Z(t) = P(Z < t) = \sum_k P(XY < t \mid X = k) \times P(X = k)$$

$$F_Z(t) = P(Z < t) = \sum_k P(kY < t) \times P(X = k)$$

where the term with $X = 0$ is treated separately:

$$F_Z(t) = P(0 < t|X = 0) \times P(X = 0) + \sum_{k \neq 0} P\left(Y < \frac{t}{k}\right) \times P(X = k)$$

$$F_Z(t) = \sum_{k \neq 0} f_X(k) F_Y\left(\frac{t}{k}\right) + P(X = 0) \qquad \text{(Equation 2)}$$

The next step is the selection of the number of summands in Equation 2. As previously assumed, $f_X(k) = P(X = k)$ is a Poisson distribution where $P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$ and the rate of convergence in the series depends solely on the rate of convergence to *0* of $\frac{\lambda^k}{k!}$ and not on *t*, whereas the second multiplier $P\left(Y < \frac{t}{k}\right) \leq 1$! Therefore, for all values of *t* and an arbitrary $\delta > 0$ there is value of *n* such that:

$$\sum_{k>n} \frac{\lambda^k e^{-\lambda}}{k!} F_Y\left(\frac{t}{k}\right) < \delta \qquad \text{(Equation 3)}$$

In our case, δ can be set, for example, to 1/1000. Thus, instead of solving the quantile equation for t_p with an infinite series, on the left-hand side of the equation we have:

$$F_Z(t) = P(Z < t) = \sum_k P\left(Y < \frac{t}{k}\right) \frac{\lambda^k e^{-\lambda}}{k!} = p \qquad \text{(Equation 4)}$$

We can then solve the equation:

$$F_Z(t, n) = \sum_{k \leq n} \frac{\lambda^k e^{-\lambda}}{k!} F_Y\left(\frac{t}{k}\right) = p \qquad \text{(Equation 5)}$$

with only *n* summands.

For example, if we choose $p = 0.95$, $\delta = 1/1000$, and *n* such that Equation 3 takes place, then the solution $t_p(n)$ of Equation 4 is such that:

$$\left| F_Z\left(t_p(n)\right) - F_Z\left(t_p(n), n\right) \right| < \frac{1}{1000} \qquad \text{(Equation 6)}$$

In other words, a quantile found from Equation 5 is almost the true value, with a resulting error precision in probability of less than 0.1%.

The only outstanding issue that remains is to find an estimate for *n* given any level of *δ*. We have:

$$\sum_{k>n} \frac{\lambda^k e^{-\lambda}}{k!} F_Y\left(\frac{t}{k}\right) < e^{-\lambda} \sum_{k>n} \frac{\lambda^k}{k!} \qquad \text{(Equation 7)}$$

The exponential series $R_n(\lambda) = \sum_{k>n} \frac{\lambda^k}{k!}$ in Equation 7 is bounded by $\frac{\lambda^{n+1} e^{\lambda}}{(n+1)!}$ by applying the Taylor's Expansion Theorem, with the remainder of the function left for higher exponential function expansions. By substituting the upper bound for $R_n(\lambda)$ in Equation 7, we have:

$$\sum_{k>n} \frac{\lambda^k e^{-\lambda}}{k!} F_Y\left(\frac{t}{k}\right) < \frac{\lambda^{n+1}}{(n+1)!} \qquad \text{(Equation 8)}$$

Now we need to find the lower bound in n for the solution of the inequality:

$$\frac{\lambda^{n+1}}{(n+1)!} < \delta \qquad \text{(Equation 9)}$$

Consider the following two cases:

If $\lambda \leq 1$, then $\frac{\lambda^{n+1}}{(n+1)!} \leq \frac{1}{(n+1)!} \leq (n+1)^{-(n+1)}e^{n}$. Consequently, we can solve the inequality $(n+1)^{-(n+1)}e^{n} < \delta$. Since n^{n} grows quickly, we can simply take $n > -\ln\delta$. For example, for $\delta = \frac{1}{1000}$, it is sufficient to set $n = 7$ to satisfy Equation 9.

If $\lambda > 1$, then, in this case, using the same bounds for the factorial, we can choose n such that:

$$(n+1)(ln(n+1) - ln\lambda - 1) > -ln\delta - 1 \qquad \text{(Equation 10)}$$

To make the second multiplier greater than 1, we will need to choose $n > e^{2+ln\lambda} - 1$.

Approximation to the solution of the equation $F_Z(t) = p$ for a quantile value

From the previous considerations we found that instead of solving $F_Z(t) = p$ for t, we can solve $F_Z(t,n) = \sum_{k\leq n}\frac{\lambda^{k}e^{-\lambda}}{k!}F_Y\left(\frac{t}{k}\right) = p$ with n set at the level indicated above. The value for t_p resulting from such a substitution will satisfy the inequality $\left|F_Z\left(t_p(n)\right) - F_Z\left(t_p(n),n\right)\right| < \delta$.

Solution of the equation $F_Z(t,n) = p$ given n and δ

By moving t to the left one unit at a time, we can find the first occurrence of the event $t = a$ such that $F_Z(a,n) \leq p$. Similarly, moving t to the right we can find b such that $F_Z(b,n) \geq p$. Now we can use a simple Bisection Method or other search algorithms to find the optimal solution to $F_Z(t,n) = p$.

Example Application

Mathematical statisticians came up with various probability distributions through the use of convolution, among other methods. For example, if there are two independent and identically distributed (*i.i.d.*) random variables, X and Y, and their respectively known probability density functions (PDF) are $f_X(x)$ and $f_Y(y)$, we can then generate a new probability distribution by combining

X and Y using basic summation, multiplication, and division; for example, the F-distribution is a division of two Chi-Square distributions, the normal distribution is a sum of multiple uniform distributions, and so on. To illustrate how this works, consider the cumulative distribution function (CDF) of a joint probability distribution between the two random variables X and Y:

$$F_{X+Y}(u) = \iint\limits_{x+y\leq u} f(x,y)\,dx\,dy = \int_{-\infty}^{\infty}\left(\int_{y=-\infty}^{u-x} f(x,y)\,dy\right)dx$$

Differentiating that CDF equation yields the PDF:

$$f_{X+Y}(u) = \int_{-\infty}^{\infty} f(x,u-x)\,dx$$

Example Application 1: The convolution of the simple sum of two identical and independent uniform distributions approaches the triangular distribution.

As a simple example, if we take the sum of two *i.i.d.* uniform distributions with a minimum of 0 and maximum of 1, we have:

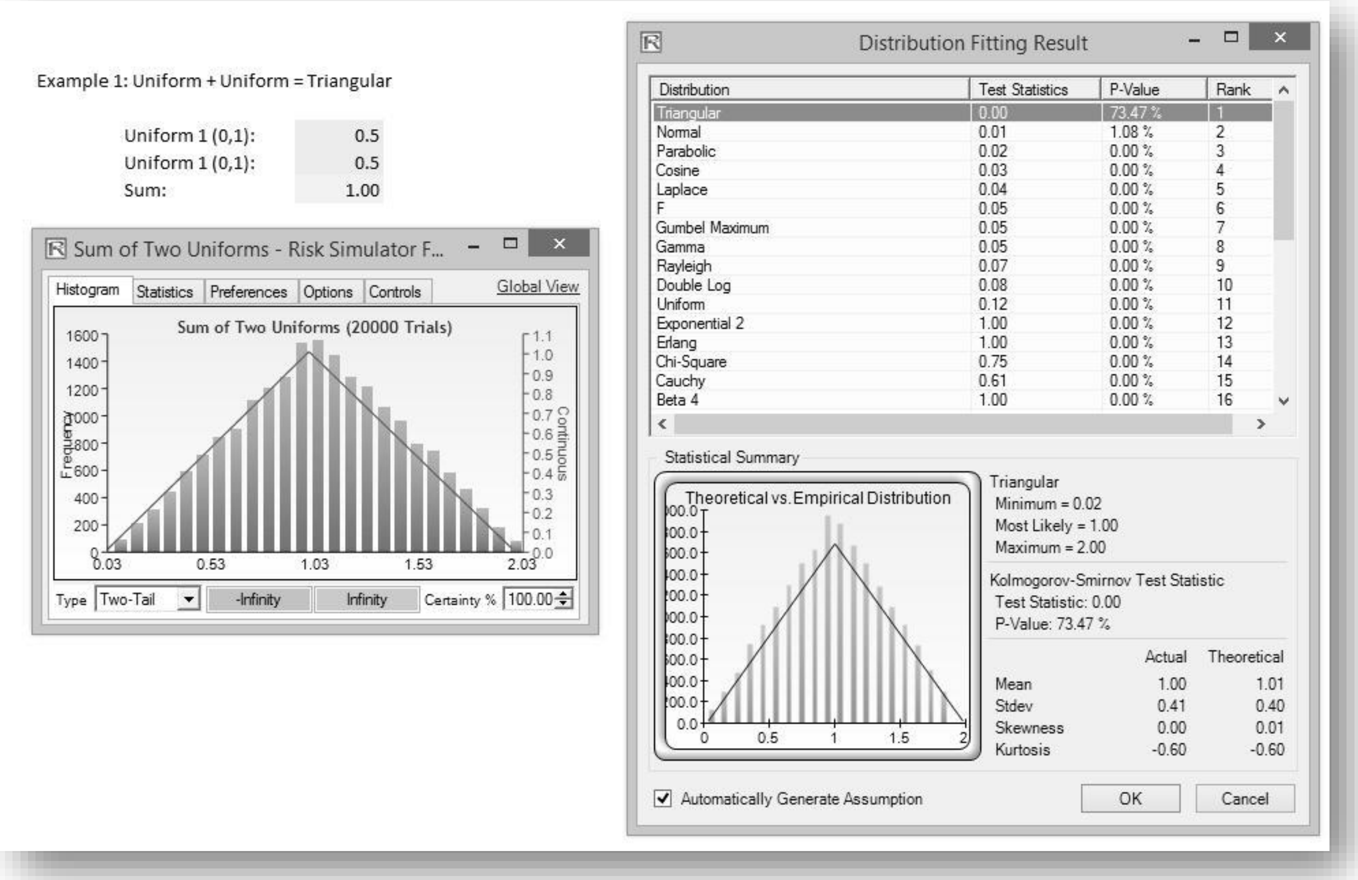
$$f_{X+Y}(u) = \int_{-\infty}^{\infty} f(x)\,f(u-x)\,dx$$

where for a Uniform [0, 1] distribution, $f(x)=1 \; when \; 0\leq x\leq 1$, we have:

$$f_{X+Y}(u) = \int_{0}^{1} f(u-x)\,dx = \int_{u-1}^{u} f(t)\,dt = \begin{cases} u & u\leq 1 \\ 2-u & 1<u\leq 2 \end{cases}$$

which approaches a simple triangular distribution.

Figure 1 shows an empirical approach where two Uniform [0, 1] distributions are simulated for 20,000 trials and their sums added. The computed empirical sums are then extracted and the raw data fitted using the Kolmogorov–Smirnov fitting algorithm (Risk Simulator software was used for this example). The triangular distribution appears as the best-fitting distribution with a 74% goodness of fit. As seen in the convolution of only two uniform distributions, the result is a simple triangular distribution.



Example 1: Uniform + Uniform = Triangular

Uniform 1 (0,1):	0.5
Uniform 1 (0,1):	0.5
Sum:	1.00

Distribution	Test Statistics	P-Value	Rank
Triangular	0.00	73.47 %	1
Normal	0.01	1.08 %	2
Parabolic	0.02	0.00 %	3
Cosine	0.03	0.00 %	4
Laplace	0.04	0.00 %	5
F	0.05	0.00 %	6
Gumbel Maximum	0.05	0.00 %	7
Gamma	0.05	0.00 %	8
Rayleigh	0.07	0.00 %	9
Double Log	0.08	0.00 %	10
Uniform	0.12	0.00 %	11
Exponential 2	1.00	0.00 %	12
Erlang	1.00	0.00 %	13
Chi-Square	0.75	0.00 %	14
Cauchy	0.61	0.00 %	15
Beta 4	1.00	0.00 %	16

Triangular
Minimum = 0.02
Most Likely = 1.00
Maximum = 2.00

Kolmogorov-Smirnov Test Statistic
Test Statistic: 0.00
P-Value: 73.47 %

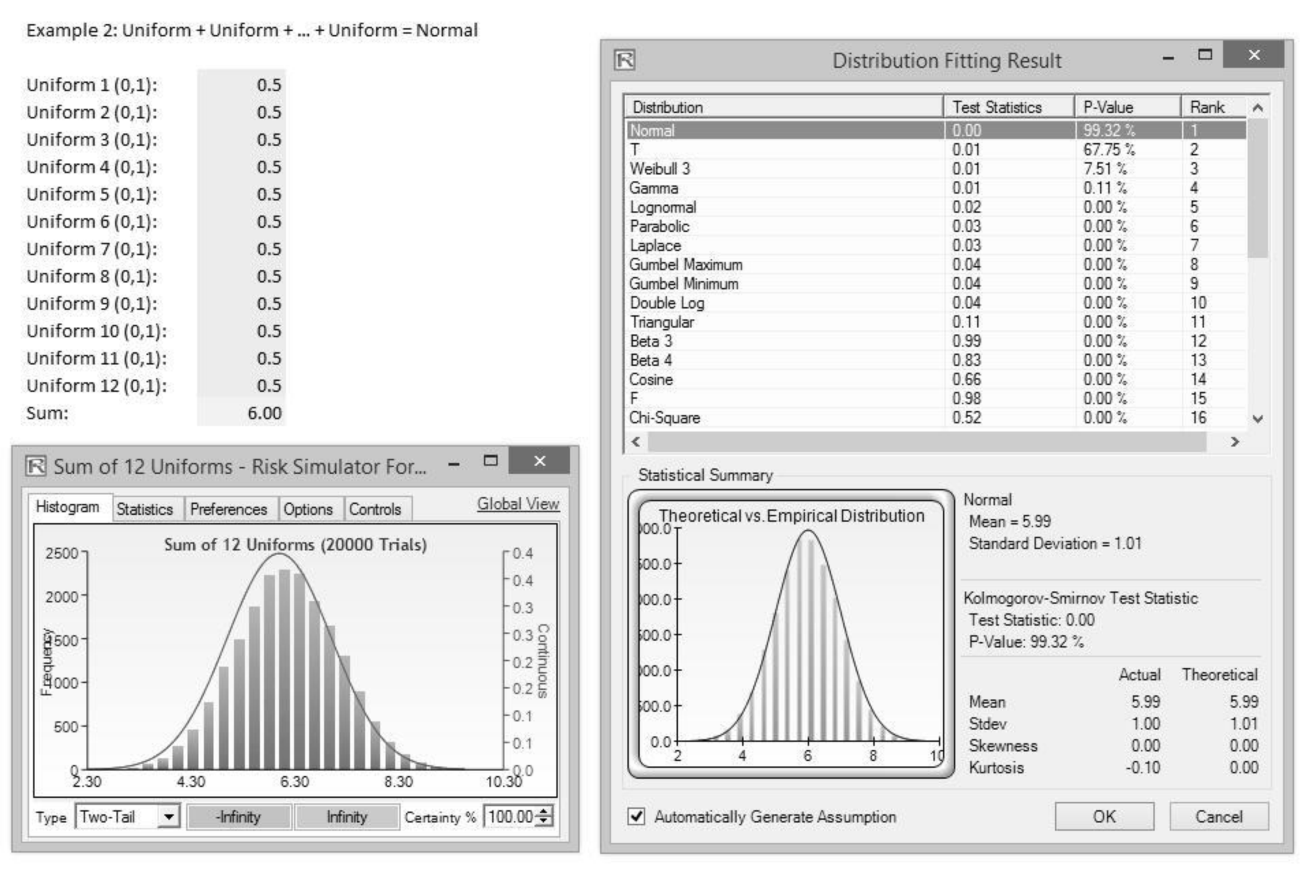
	Actual	Theoretical
Mean	1.00	1.01
Stdev	0.41	0.40
Skewness	0.00	0.01
Kurtosis	-0.60	-0.60

☑ Automatically Generate Assumption

Figure 1: Convolution of Two Uniform Distributions via Simulation

Example Application 2: The convolution simple sum of 12 identical and independent uniform distributions approaches the normal distribution.

If we take the same approach as used in Example Application 1 and simulate 12 *i.i.d.* Uniform [0, 1] distributions and sum them, we would obtain a very close to perfect normal distribution as shown in Figure 2, with a goodness of fit at 99.3% after running 20,000 simulation trials.


Example 2: Uniform + Uniform + ... + Uniform = Normal

Uniform 1 (0,1):	0.5
Uniform 2 (0,1):	0.5
Uniform 3 (0,1):	0.5
Uniform 4 (0,1):	0.5
Uniform 5 (0,1):	0.5
Uniform 6 (0,1):	0.5
Uniform 7 (0,1):	0.5
Uniform 8 (0,1):	0.5
Uniform 9 (0,1):	0.5
Uniform 10 (0,1):	0.5
Uniform 11 (0,1):	0.5
Uniform 12 (0,1):	0.5
Sum:	6.00

Distribution	Test Statistics	P-Value	Rank
Normal	0.00	99.32 %	1
T	0.01	67.75 %	2
Weibull 3	0.01	7.51 %	3
Gamma	0.01	0.11 %	4
Lognormal	0.02	0.00 %	5
Parabolic	0.03	0.00 %	6
Laplace	0.03	0.00 %	7
Gumbel Maximum	0.04	0.00 %	8
Gumbel Minimum	0.04	0.00 %	9
Double Log	0.04	0.00 %	10
Triangular	0.11	0.00 %	11
Beta 3	0.99	0.00 %	12
Beta 4	0.83	0.00 %	13
Cosine	0.66	0.00 %	14
F	0.98	0.00 %	15
Chi-Square	0.52	0.00 %	16

Statistical Summary

Normal
Mean = 5.99
Standard Deviation = 1.01

Kolmogorov-Smirnov Test Statistic
Test Statistic: 0.00
P-Value: 99.32 %

	Actual	Theoretical
Mean	5.99	5.99
Stdev	1.00	1.01
Skewness	0.00	0.00
Kurtosis	-0.10	0.00

☑ Automatically Generate Assumption

Figure 2: Convolution of 12 Uniform Distributions to Create a Normal

Example Application 3: The convolution simple sum of multiple identical and independent exponential distributions approaches the gamma (Erlang) distribution.

In this example, we sum two *i.i.d.* exponential distributions and generalize it to multiple distributions. To get started, we use two identical Exponential [$\lambda = 2$] distributions:

$$f_{X+Y}(z)=\int_{0}^{Z} f_X(x)\,f_Y(z-x)\,dx=\int_{0}^{Z} \lambda e^{-\lambda X}\lambda e^{-\lambda(Z-X)}\,dx=\lambda^2 ze^{-\lambda Z}$$

where $f(x)=\lambda e^{-\lambda X}$ is the PDF for the exponential distribution for all $x \geq 0; \lambda \geq 0$, and the distribution's mean is $\beta = 1/\lambda$.

If we generalize to n random *i.i.d.* exponential distributions and apply mathematical induction:

$$f_{X_1+X_2+\ldots+X_n}(x) = \frac{x^{n-1}e^{-x/\beta}}{(n-1)!\beta^n} = \Gamma[0,n,1/\lambda]$$

$$f(x) = \frac{x^{\alpha-1}e^{-x/\beta}}{\Gamma(\alpha)\beta^{\alpha}} \quad \text{with any value of } \alpha > 0 \text{ and } \beta > 0$$

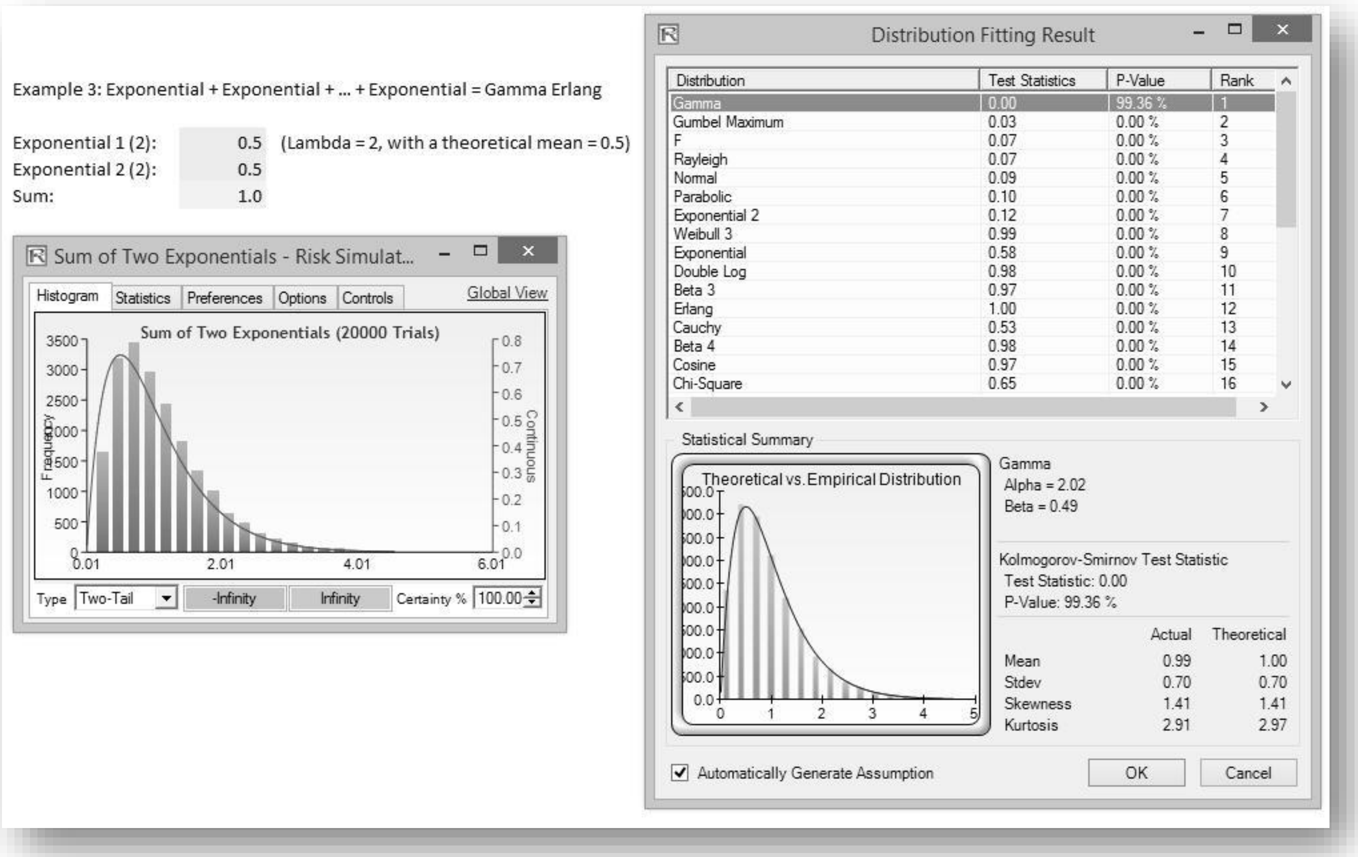
This is, of course, the generalized gamma distribution with a and β for the shape and scale parameters:

$$f_{X_1+X_2+\ldots+X_n}(x) = \Gamma[0,n,1/\lambda] = \Gamma[0,\alpha,\beta]$$

When the β parameter is a positive integer, the gamma distribution is called the Erlang distribution, used to predict waiting times in queuing systems, where the Erlang distribution is the sum of random variables each having a memoryless exponential distribution. Setting n as the number of these random variables, the mathematical construct of the Erlang distribution is:

$$f(x) = \frac{x^{\alpha-1}e^{-x}}{(\alpha-1)!} \quad \text{for all } x > 0 \text{ and all positive integers of } a$$

The empirical approach is shown in Figure 3, where we have two exponential distributions with $\lambda = 2$ (this means that the mean $\beta = 1/\lambda = 0.5$). The sum of these two distributions, after running 20,000 Monte Carlo simulation trials and extracting and fitting the raw simulated sum data (Figure 3), shows a 99.4% goodness of fit when fitted to the gamma distribution where the $a = 2$ and $\beta = 0.5$ (rounded), corresponding to $n = 2$ and $\lambda = 2$.



Example 3: Exponential + Exponential + … + Exponential = Gamma Erlang

Exponential 1 (2):	0.5	(Lambda = 2, with a theoretical mean = 0.5)
Exponential 2 (2):	0.5	
Sum:	1.0	

Sum of Two Exponentials - Risk Simulat...

Type: Two-Tail

Certainty %: 100.00

Distribution Fitting Result

Distribution	Test Statistics	P-Value	Rank
Gamma	0.00	99.36 %	1
Gumbel Maximum	0.03	0.00 %	2
F	0.07	0.00 %	3
Rayleigh	0.07	0.00 %	4
Normal	0.09	0.00 %	5
Parabolic	0.10	0.00 %	6
Exponential 2	0.12	0.00 %	7
Weibull 3	0.99	0.00 %	8
Exponential	0.58	0.00 %	9
Double Log	0.98	0.00 %	10
Beta 3	0.97	0.00 %	11
Erlang	1.00	0.00 %	12
Cauchy	0.53	0.00 %	13
Beta 4	0.98	0.00 %	14
Cosine	0.97	0.00 %	15
Chi-Square	0.65	0.00 %	16

Statistical Summary

Gamma
Alpha = 2.02
Beta = 0.49

Kolmogorov-Smirnov Test Statistic
Test Statistic: 0.00
P-Value: 99.36 %

	Actual	Theoretical
Mean	0.99	1.00
Stdev	0.70	0.70
Skewness	1.41	1.41
Kurtosis	2.91	2.97

☑ Automatically Generate Assumption

Figure 3: Convolution of Exponentials to Create a Gamma Erlang

Analytical Framework Approach

An analytical framework approach is used if cross-sectional data can be gathered. Specifically, data on measurable outputs, such as those in a production function:

$$\text{Production function } Y = f(\epsilon, \tau, \varphi, \theta, \omega, \ldots, \varepsilon) \qquad \text{(Equation 11)}$$

where Y is the measurable production output, ϵ is the education and training investment amount, τ is the technology supporting said production, φ is the capital investment, θ is the organizational design structure, ω the environmental impacts, and ε is the forecast error in the model. Therefore, we can determine $\frac{\partial Y}{\partial \epsilon}$, and this will represent the expected change in average value of production with respect to each unitary change in educational investment, after accounting for all the other variables. In other words, this is the net effect of educational contribution to overall outcomes.

Performing some partial differentials, we obtain:

$$\frac{\partial Y}{\partial \epsilon} = \frac{\partial f}{\partial \tau}\frac{\partial \tau}{\partial \epsilon} + \frac{\partial f}{\partial \varphi}\frac{\partial \varphi}{\partial \epsilon} + \frac{\partial f}{\partial \theta}\frac{\partial \theta}{\partial \epsilon} + \frac{\partial f}{\partial \omega}\frac{\partial \omega}{\partial \epsilon} \qquad \text{(Equation 12)}$$

A nonlinear regression can be run on the above assuming continuous data variables, or Logit, Probit, and Tobit models can be run on discrete and truncated limited dependent variables (Mun, 2016).

Example Application

As an example, suppose we run an experiment of performing an intensive three-month sales and marketing training of a sales-oriented software company. The sales output *Y* at time *t* is modeled by:

$$Y_t = \beta_0 + \beta_1\epsilon_{t-1} + \beta_1\tau_{t-1} + \beta_1\varphi_{t-1} + \beta_1\theta_{t-1} + \varepsilon \qquad \text{(Equation 13)}$$

yielding the following results (based on a sample simulated set of notional data):

$$Y_t = 25.5 + 2.35\epsilon_{t-1} + 1.25\tau_{t-1} + 3.5\varphi_{t-1} + 1.5\theta_{t-1} + \varepsilon \qquad \text{(Equation 14)}$$

All variables are significant at an $a = 0.05$, adjusted $R^2 = 0.85$. This means that $\frac{\partial Y}{\partial \epsilon} = 2.35$, where for each dollar of education invested, we have a \$2.35 net return, providing an ROI of 135%.

Empirical Impact Approach

The empirical impact approach can be used to determine if there is, indeed, statistically significant value add existing in post-training compared to situations without any training. If the standard deviations of these two sample datasets (with and without the requisite training and education) are still unknown but assumed to be different, combining them into a single pooled estimate as done previously would be inappropriate (Mun, 2016). Therefore, the sample standard deviations (s) will be used independently to estimate the population standard deviations (σ).

Nonetheless, normality of the underlying dataset is assumed, although this assumption becomes less important with larger datasets. The two-sample unequal variance t-test would be needed, and its specifications are described below:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\bar{\mu}_1 - \bar{\mu}_2)}{\sqrt{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}} \text{ and } df = \frac{\left(s_1^2/n_1 + s_2^2/n_2\right)^2}{\frac{\left(s_1^2/n_1\right)^2}{n_1 - 1} + \frac{\left(s_2^2/n_2\right)^2}{n_2 - 1}} \qquad \text{(Equation 15)}$$

H_0: $\mu_1 = \mu_2$, that is, the two samples' means are statistically similar.

In addition, if the collected data is limited and categorical or ordinal in nature, or if there are significant biases in the data, we can apply the Kruskal–Wallis test, which is an extension of the Wilcoxon Signed-Rank test by comparing more than two independent samples. The corresponding parametric test is the One-Way ANOVA, but unlike the ANOVA, the Kruskal–Wallis does not require that the dataset be randomly sampled from normally distributed populations with equal variances. The Kruskal–Wallis test is a two-tailed hypothesis test where the null hypothesis is such that the population medians of each treatment are statistically identical to the rest of the group; that is, there is no effect among the different treatment groups. Similar to the ANOVA method, the Kruskal–Wallis tests the following hypotheses:

$H_0: m_1 = m_2 = \cdots = m_k \;\; for\ i = 1\ to\ k$ (population medians are identical).

The method starts off with *k* variables to be tested. For each variable, the data are ranked from smallest to largest, with the smallest value receiving the rank of 1, and all tied ranks are assigned their average values. Then, all the ranks are summed for each variable, yielding a list of summed ranks $\Sigma(R1), \Sigma(R2), \ldots, \Sigma(RK)$. Then, the *H* statistic is computed using:

$$H = \frac{12}{N(N+1)}\left[\frac{(\sum R_1)^2}{n_1} + \frac{(\sum R_2)^2}{n_2} + \ldots + \frac{(\sum R_k)^2}{n_k}\right] - 3(N+1) \qquad \text{(Equation 16)}$$

The calculated *H* is compared to critical *H* values computed using a chi-square distribution with degrees of freedom $df = k - 1$.

Work Lifecycle Approach

Finally, the work lifecycle approach can be used to determine the lifecycle valuation of education. According to Kamarck, Thie, Adelson, and Krull (2010), several past studies of individuals with privately funded education such as a master of business administration (MBA) or other technical master's degree, show that they earn an average rate of return of at least "46% more than a bachelor's degree in a 2008 study… and the ROI ranges between 27% to 36% for an MBA compared to some other technical master's degree."

However, the application of a similar methodology might not work well within the DoD because the U.S. military's human resource environment is such that it is a closed internal and hierarchical structure. For instance, an officer's pay is based on his or her rank and years of service, regardless of educational background. It can be argued that higher education may result in higher efficiency and productivity, thereby increasing the speed of promotions, but these are fairly difficult to quantify. An alternate approach might be to consider the years of service beyond the time the education was received. This amounts to the value of retention, in other words, how much the military can save in costs by having a higher retention and reutilization rate than having to train a new officer to replace a billet due to attrition.

The model might look something like:

$$ROI = \frac{\Psi[f(h, \tau_t, o_t) + \delta P_t(V_t)] - C_0}{C_0}$$

where Ψ is the years of service; C_0 is the cost of education; δP_t is the change in productivity due to the new knowledge gained (with a nonlinear depreciation over time); V_t is the salary and overhead cost of the billet; τ_t is the learning curve measured in time to train a new officer to adequately replace the outgoing officer; and o_t is the opportunity cost of lower retention rate or cost of the attrition. With the proper experimental approach, these variables can be adequately measured to provide a robust ROI measure.

As a matter of comparison, for privately funded educational programs, one can much more easily model the ROI where we can use a traditional net present value (NPV) to determine the ROI, such as:

$$NPV = \sum_{i=n+j}^{k-j-n} \left[S_e \pi_{e_t} - S_0 \pi_{0_t}\right] e^{-rt} - \sum_{i=1}^{j} C_t e^{-rt}$$

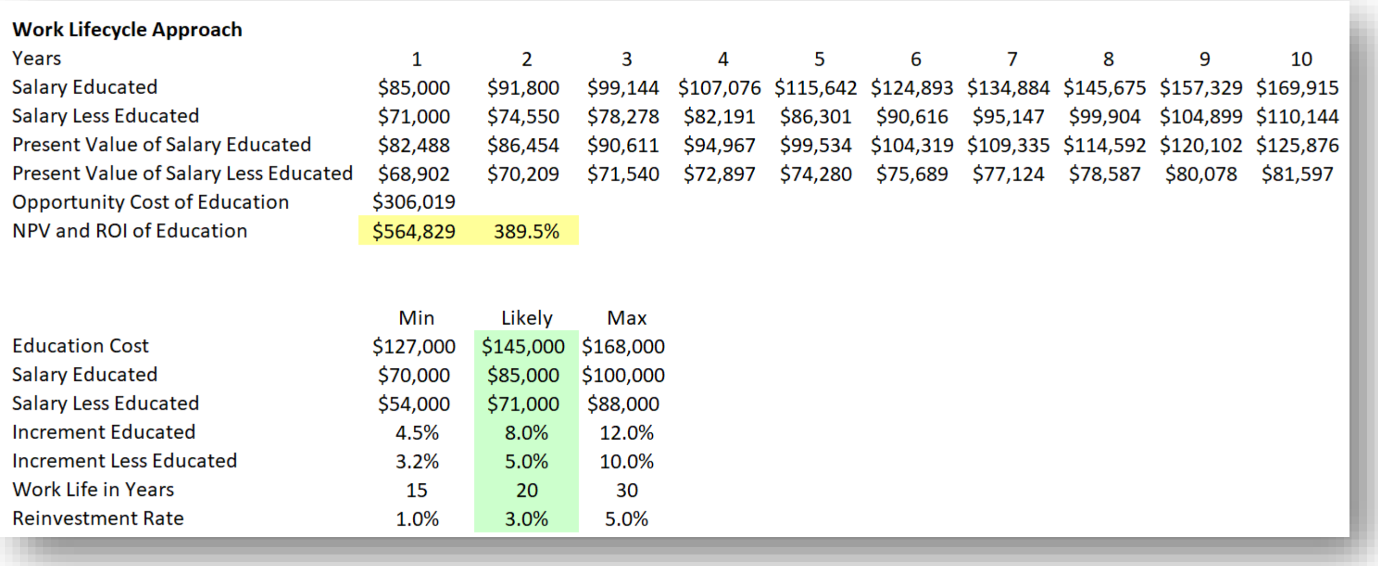
$$ROI = \frac{\sum_{i=n+j}^{k-j-n} \left[S_e \pi_{e_t} - S_0 \pi_{0_t}\right] e^{-rt} - \sum_{i=1}^{j} C_t e^{-rt}}{\sum_{i=1}^{j} C_t e^{-rt}}$$

where S_e is the salary with the education; S_0 is the presumably lower salary without the requisite education; π is the inflationary and natural growth rate of the salary over time t, each with a different acceleration slope for educated e and uneducated *0* rates; r is the reinvestment rate or opportunity cost of the cost of education C_t which changes over time, over the course of the education j; and the analysis is performed on the lifecycle of the individual's working life, starting from the current age n to the retirement age k (the age of natural attrition, retirement age, or average age of leaving the employment market). These inputs can be Monte Carlo risk simulated using the integrated risk management approach, as will be discussed in Section IV.

Example Application

As an example, according to various job and educational websites (Classroom, 2020), an MBA in business earns an average nationwide starting salary of $70,000 to$100,000 depending on the field, whereas an undergraduate degree holder in business earns $54,000 to$88,000. The average cost of the MBA is between $127,000 and$168,000 for a two-year study program. With a prevailing savings rate and other low-risk investment returns of between 1% and 5% annually, and assuming that the average working life is approximately 15–30 years, we compute the ROI and apply Monte Carlo risk simulation to these inputs to obtain an ROI estimate (Figure 4). Simulation is required because we cannot say for sure a computed ROI is, indeed, the representative value that applies in all cases to all individuals.

After running 100,000 simulation trials, we determine for those having an MBA that, given the sample representative input assumptions, there is a 77% probability (Figure 5), or over three-quarters, it pays off, with a positive ROI. Of course, given the economic conditions and salary assumptions, there is still a 23% probability (almost one quarter of those with an MBA) that the added cost and time required to obtain an MBA is not worth it, yielding an economic return that is negative. Figure 6 shows that the median ROI is 318% and the 50% confidence interval (also known as the interquartile range, where half the population falls within this range, and we ignore the outliers such as those who made millions and are promoted to C-level executive ranges, or those who voluntarily leave the workforce for personal or family reasons) has an ROI between 25% on the worst case (those working in lower-paying jobs and industries) and 698% in the best case (those living in higher cost of living locations working in an expansionary economic environment). The results indicate that having an MBA provides a return that is 1.25× to 8× of the education cost. The rough order magnitude computations shown here roughly parallel the third-party studies referenced by the RAND research.

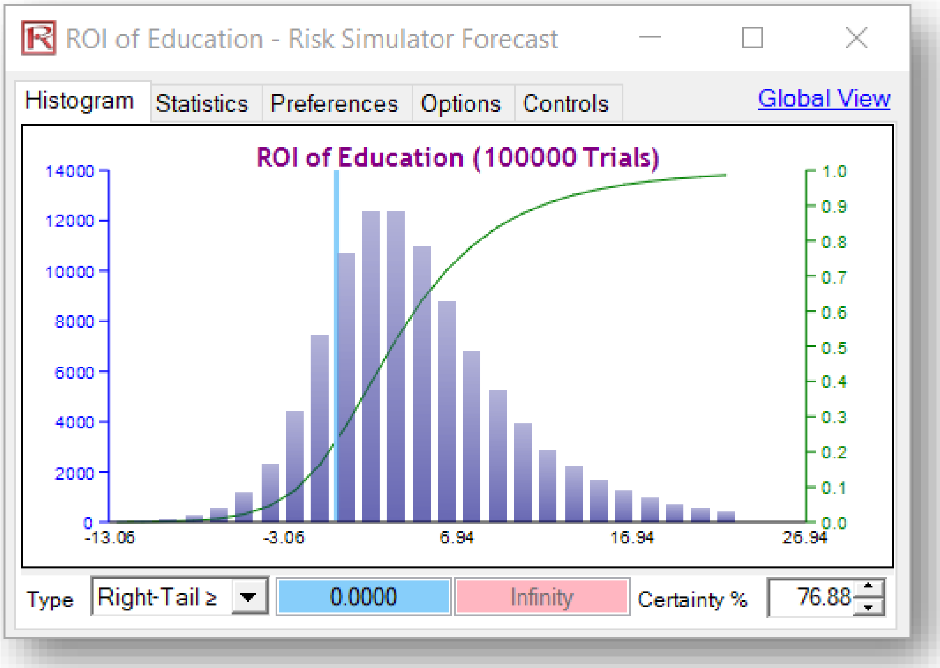


Work Lifecycle Approach

Years	1	2	3	4	5	6	7	8	9	10
Salary Educated	$85,000	$91,800	$99,144	$107,076	$115,642	$124,893	$134,884	$145,675	$157,329	$169,915
Salary Less Educated	$71,000	$74,550	$78,278	$82,191	$86,301	$90,616	$95,147	$99,904	$104,899	$110,144
Present Value of Salary Educated	$82,488	$86,454	$90,611	$94,967	$99,534	$104,319	$109,335	$114,592	$120,102	$125,876
Present Value of Salary Less Educated	$68,902	$70,209	$71,540	$72,897	$74,280	$75,689	$77,124	$78,587	$80,078	$81,597
Opportunity Cost of Education	$306,019									
NPV and ROI of Education	$564,829	389.5%								

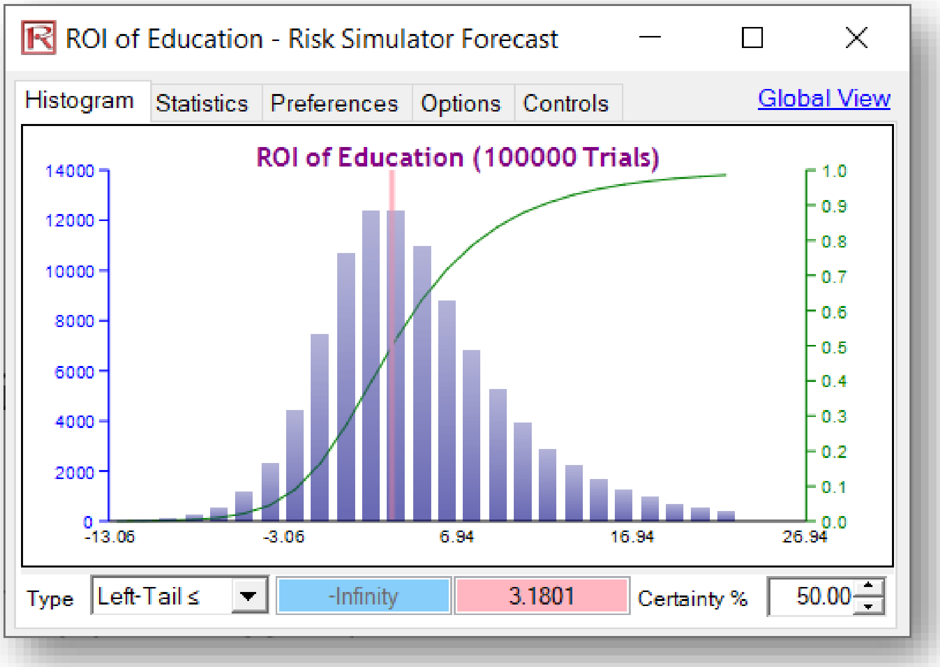
	Min	Likely	Max
Education Cost	$127,000	$145,000	$168,000
Salary Educated	$70,000	$85,000	$100,000
Salary Less Educated	$54,000	$71,000	$88,000
Increment Educated	4.5%	8.0%	12.0%
Increment Less Educated	3.2%	5.0%	10.0%
Work Life in Years	15	20	30
Reinvestment Rate	1.0%	3.0%	5.0%

Figure 4: Work Lifecycle Model and Simulation Assumptions for 30 Years



Type Right-Tail ≥ 0.0000 Infinity Certainty % 76.88

Figure 5: Probability of Positive ROI



Type Left-Tail ≤ -Infinity 3.1801 Certainty % 50.00

Figure 6: Median ROI

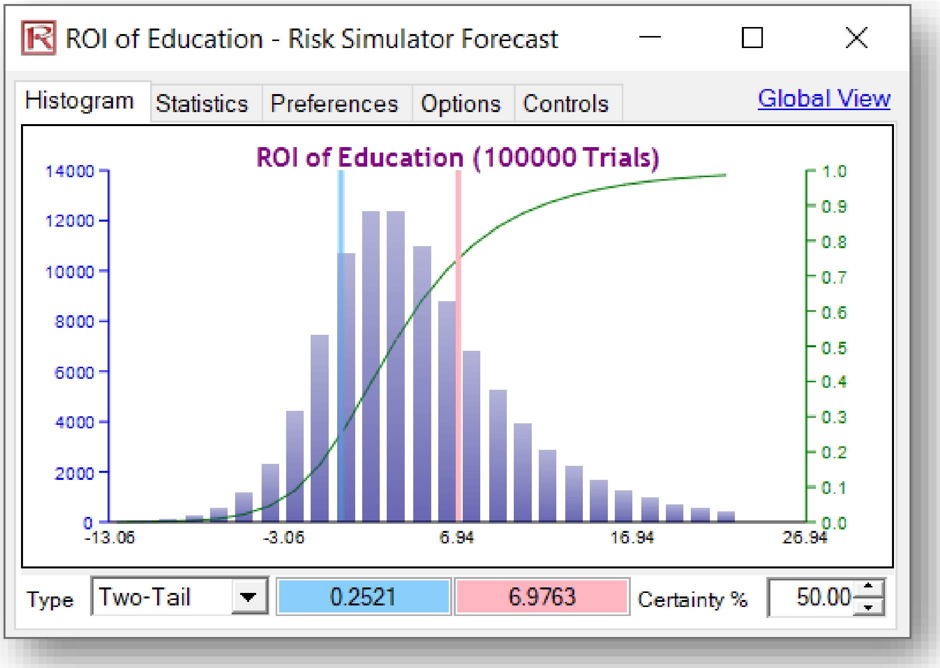


Figure 7: IQR 50% Confidence Interval of ROI

Intrinsic and Intangible Value Proposition of Education for the Fleet

Intangible and intrinsic value exists in both military education and research but cannot be readily quantified in any standard ROI calculations. In nonmilitary college education in the private sector, higher education brings with it many intangible value add, such as value to society (Blagg and Blom, 2018) through diversification and innovation of the nation's economy, encourage graduates to be more civic minded, increased wages and lower crime rate, increase tax receipts of the country, increased productivity and output, lower expenditures on policing due to lower crime, and lowers dependencies on social welfare programs.

The intangible value in military education is slightly different. The military is a closed vertical society. For instance, in the E4S report, a survey of past Naval students at NPS, NWC, and USNA indicated that approximately 96% agreed that formal education was extremely useful or very useful in their naval careers. In fact, the study found that military personnel have more positive perceptions of their institutions than civilian personnel. In addition, the faculty at USNA perceives their institution as "better at preparing naval officers to be more effective leaders, excel in their fields of study, apply their education to real world situations, establish and manage effective teams, and understand critical strategies significantly better," (Department of the Navy, 2018) than if they attended civilian institutions.

We can certainly conclude that the intangible value of military education is significant, in developing leadership and critical thinking skills for junior as well as senior officers. The military-oriented curriculum taught by faculty members with former military experience or knowledge allows the flow of institutional knowledge down to the students. Although these intangible and qualitative side of military education is significant, this current research focuses on the more quantitative measure of ROI. Nonetheless, the creative thinking, leadership, strategic thought, and quick tactical decision-making skills can be honed through education, especially when taught by a faculty with military-based academic and research backgrounds.

Education is a lifelong pursuit, and it is no different for naval officers. The strategic, tactical, and innovative changes and challenges in the future requires continuous education of our joint forces in order to maintain a competitive advantage over our current and future adversaries.

In addition, official naval policy in the DON dictates that naval officers should obtain a graduate degree. The Goldwater Nichols reforms requires some form of professional military education, and promotion and selection board convening officers usually take graduate degrees into serious consideration when reviewing an officer's career advancement.

Current and past senior leadership in the Navy sees education as critical to the Fleet's future warfighting capacity, for instance:

"Education is critical to the future of the Nation's warfighting capacity, just as much as… augmenting their talents with the very best platforms and technologies," (Admiral Mike Mullen, 17th Chairman of the Joint Chiefs of Staff, 28th Chief of Naval Operations).

"In the end, 21st century warfare is brain-on-brain conflict, and we must build our human capital and intellectual capacity as surely as we produce the best warfighting technology if we are going to win the nation's wars and advance its security," (Admiral James Stavridis, former Supreme Allied Commander Europe).

"The proven power of combining shared education with deep operational experience as a way of preparing Navy leaders for profound changes in strategic and technologic direction is part of our history… By seizing this moment, the DON can synchronize the realignment of its education organization and its talent management process in ways that will accelerate service-wide mastery of the changing national security environment," (VADM Patricia Traccey, Former OPNAV N7 and Director, Navy Staff).

IV. ENHANCED METHODOLOGIES

Whereas the previous section outlined some theoretical approaches, this section details additional enhanced methods culled from data science and uncertainty-based decision analytics, namely, knowledge value added and integrated risk management, where all these methodologies, theoretical and enhanced, can be combined in various ways to create a robust set of methodologies to triangulate the true ROI of military education and research.

Knowledge Value Added

Benefits

KVA is an objective, quantifiable method to measure the value associated with a system and the subprocesses within the system. The value measurements of each process are ratio-scale numbers, allowing analysts to compare them with the values from other subprocesses to determine their relative effectiveness. PMs can determine the value generated from the human component against the value added by IT processes. Because of the scales, PMs can use these measurements to develop useful ratios in their analysis of the program's performance. Productivity ratios such as ROK, output of a process divided by the process cost, and ROI, output minus cost divided by cost, can be adapted for use in KVA. The ROKs and ROIs, which are always 100% correlated, give managers information about the amount of value a process generates compared to the amount of money spent to create the value. Unlike any other methodology, KVA assigns these figures to both the process and subprocesses rather than only the process as a whole.

Conducting an analysis of a program using KVA will give a PM a clearer picture of the operational components of the program. While organizations likely have metrics used to determine the performance of a project or operation, ROK will give them additional information to improve their management decisions. PMs can determine the relative value of the components that comprise the program. Knowing a particular job or sub-process gives the same output value as a different process but at a different cost may provide context for the performance of the system. This, in turn, gives experienced managers the information needed to allocate resources to specific components of a program that need improvement or should be utilized more frequently.

While a KVA analysis can provide information that will change the course of a program or project, it does not require significant changes to organizational structure or reporting processes to do so. The evaluation can be conducted during normal operating conditions without introducing complicated new metrics into the system. Learning time, process description, or the binary query method are all based on information that should be available within the organization. A small amount of hands-on measurement may be required to verify the accuracy of the given data. As such, the analysis can be done quicker than other methodologies, giving PMs access to actionable information more rapidly.

Challenges

KVA will give analysts a quantifiable, ratio-scale number for the value of the sub-processes. However, it does this only with processes that consist of known a priori outputs. The intangible items, such as creativity and imagination, that occur within the human brain cannot be quantified with this method. In fact, no current system is able to accurately quantify these types of intangibles within a process because there is no algorithm for creativity. These factors are not common to the average user and as such, cannot be defined via any of the KVA methods- learning time, binary query, or process description—because the creativity process cannot be learned or described. However, this was only possible after the system was completed and described. KVA will assign the value of process but it cannot predict the value of potential outputs, only those that are specified a priori.

Although KVA will provide ratio-scale numbers to aid in evaluating processes within a program, the ratios are often only valid for comparisons within the same analysis. Benchmarking the raw numbers with other organizations or with different divisions in the same organization may not provide a usable assessment depending on the techniques used when determining the ROK. Another analyst may have used the binary query or process description methods to describe the outputs in an equally defensible evaluation. These numbers will not be comparable to the numbers from the learning time method unless they are normalized, even though the final analysis will result in the same relative quantitative comparisons of productivity. Other variances may cause the same issues, such as if an analyst includes the underlying infrastructure or common training for all personnel. Because these can be treated as constants across all processes they can be excluded without skewing the final results. Nevertheless, the final results of any properly conducted analysis will return the same ROKs, which is the ultimate goal of KVA.

Integrated Risk Management

IRM is a system developed by the author designed to provide management the ability to analyze risk associated with the development of a new project or initiative. IRM combines several commonly accepted analytical procedures, such as predictive modeling, Monte Carlo simulation, real options analysis, and portfolio optimization, into a single, comprehensive methodology. The methodology uses existing techniques and metrics such as discounted cash flow, ROI, and other metrics within the analytical processes to improve the traditional manner of evaluating potential projects within a company or the DoD. In contrast to the other methodologies, IRM focuses on the risk involved with a decision. It seeks to mitigate negative effects from risk while maximizing rewards from potential outcomes. At its core, IRM is a technique to provide managers the best analytic information available to use during the real options process.

There are eight steps within the IRM methodology:

1. Qualitative management screening
2. Forecast predictive modeling
3. Base case static modeling
4. Monte Carlo risk simulation
5. Real options problem framing
6. Real options valuation and modeling
7. Portfolio and resource optimization
8. Reporting, presenting, and updating analysis

While each of the individual steps provides value to a project manager, incorporating all of them in a contiguous approach will allow decision makers the most effective use of the IRM process.

Figure 8 illustrates the comprehensive IRM process. The process begins with a qualitative management screening of potential projects, assets, and initiatives that could benefit the organization. These potential additions to a company's portfolio should align with the overall strategy, mission, and goals of the company (Mun, 2016). The risks to an organization must be identified and addressed for decision makers to have a realistic picture of the challenges the projects may face (Mun, 2016). This step is not unique to IRM. Prior to a firm beginning any venture, senior leadership should ensure that the ventures they are funding are realistic options based on their expertise and vision. If these are not in alignment, the initiatives will almost certainly fail. However, by evaluating the suitability of the projects and programs at the outset, management can eliminate potential programs that are incompatible prior to additional costly analysis.

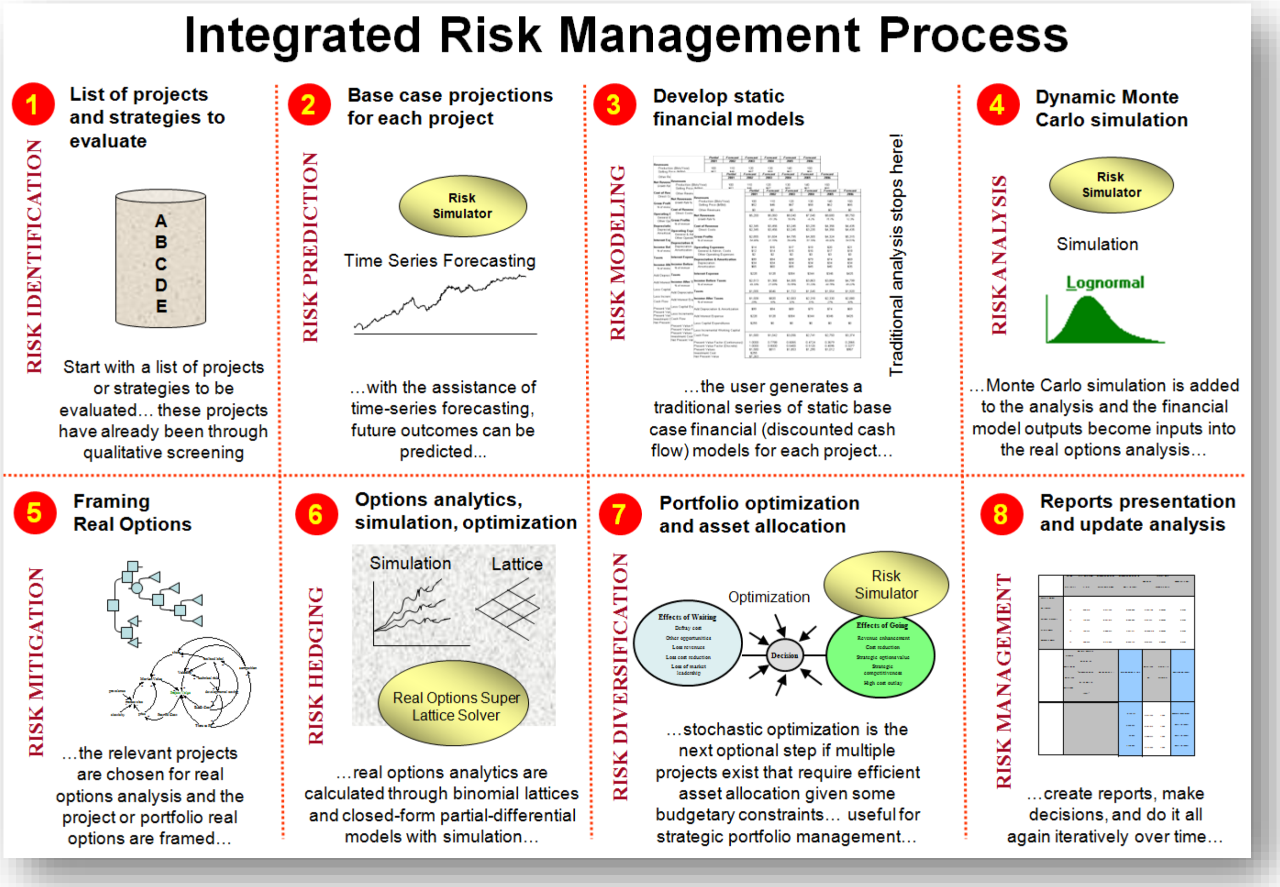


Figure 8: Integrated Risk Management process

Source: Mun (2016)

The second step is to forecast results using predictive modeling. Ideally, management will have access to historical data to use during this evaluation. Using comparable data from similar firms or projects is an acceptable alternative when the historical information is not available. With the data, analysts will use techniques such as multivariate regression analysis, time-series analysis, and others to predict a project's performance (Mun, 2016). If the data are unavailable, qualitative forecasting methods and SME estimates can be substituted for the historical or comparable information (Mun, 2016). The qualitative techniques can vary from assumptions about the growth rate to expert opinions, subjective estimates, and the Delphi method (Mun, 2016). In both cases the techniques are forecasting value and cost drivers within the project (e.g., quantity, volume, production, revenue, cost, schedule, etc.; Mun, 2016). In a nonprofit context such as the DoD acquisition lifecycle, surrogates should be used for revenue. The metrics that will define the value of a project can be projected in this analysis in place of for-profit financial measurements.

Using the results from the forecasting step, a model of discounted cash flow or similar models with a future projection of cost and benefit is created for each project, which serves as the base case analysis for future decisions (Mun, 2016). The net present value (NPV) or other ROI for the initiative is calculated via the traditional method, that is, projecting both revenue and cost and discounting the net value at an appropriate rate adjusted for standard financial risks (Mun, 2016). Additional profitability, productivity, and cost-benefit metrics, such as other variations of ROI, are calculated during this phase (Mun, 2016). The DoD and other nonprofit organizations do not collect revenue, making the profitability ratios listed meaningless without a surrogate for revenue. (KVA offers this surrogate in the form of value. Using KVA as the base case analysis allows a quantitative, common-units comparison of nonprofit projects in the same manner as a traditional revenue-generating industry.)

Next, the analyst will conduct a Monte Carlo risk simulation to obtain a better assessment of the potential risks and value of the proposed venture. While the base case static model developed in step three is a useful tool, it is based on static information and, as such, produces a single-point estimate (Mun, 2016). The information gleaned from the model may not be accurate due to the uncertainty and risks involved in future cash flows (Mun, 2016). Since financial problems inherently contain uncertainty of some form, a model that accounts for this uncertainty is necessary (Brandimarte, 2014). The Monte Carlo simulation will increase confidence in the value of a project by using statistical analysis to give a probability of ranges for different variables.

Monte Carlo simulation, or probability simulation, is a technique used to understand the impact of risk and uncertainty in financial, project management, cost, and other forecasting models (Mun, 2016). In a Monte Carlo simulation, analysts generate random scenarios and gather relevant statistics to assess situations that are affected by uncertainty (Brandimarte, 2014). Using historical data and the opinions of SMEs, analysts can input a range of possible values to simulate potential future outcomes (Mun, 2016). Since the input variables are given in a range of estimates, the model's outputs will also be a range indicating the likelihood of the possibilities. (Mun, 2016). The Monte Carlo simulation can also be run using only historical data and the computer will make a custom distribution of the variables to produce its output or with a prescribed probability distribution (Mun, 2015). In IRM, the analyst will set NPV or any of the computed ROI variations as the resulting variable(s) and run the Monte Carlo simulation thousands of times, adjusting each of the other variables to predict a range and probability of potential NPVs for the project (Mun, 2015).

The quantitative data gleaned from the Monte Carlo simulation is only useful if it provides decision makers with improved information to make decisions. The information must be converted into actionable intelligence (Mun, 2016). While the statistical analysis and other preceding steps are important, the crux of the IRM methodology is the real options assessment. To begin that process, leaders must conduct real options problem framing, step five in the IRM methodology. Real options allow managers to hedge, value, and take advantage of risks, reducing the potential downside while maximizing potential gains from volatile projects (Mun, 2016). By framing the problem through a real options lens, an organization's leadership can generate a strategic plan for the problem from several options, (Mun, 2016). Analysts will then examine chosen options in more detail (Mun, 2016).

Real options provide investors the ability to adjust the course of previous decisions based on the performance of the investment to date. They allow management to make "better and more informed strategic decisions when some levels of uncertainty are resolved through the passage of time, actions, and events" (Mun, 2015, p. 438). Options are opportunities for a company; they have a right to conduct an action without the obligation to take the future action (Dixit & Pindyck, 1995). There are several types of options and the number of names of available options varies depending on the literature source.

Generating coherent and concise reports detailing the analysis is the eighth, and final, step in IRM (Mun, 2016). If decision makers do not understand the complicated procedures that led to the investment recommendations, they will not trust the results enough to follow those recommendations (Mun, 2016). Transforming the "black-box set of analytics into transparent steps" is vital to ensuring leadership has the best possible information with which to make decisions for the company's project portfolio (Mun, 2016, p. 95). Although this is the final step within the IRM process, as additional information becomes available and the uncertainty and risk are reduced or resolved, analysts should revisit the models with updated information (Mun, 2016). Reworking the original models with the new data allows managers to make midcourse corrections to improve the performance of both the individual project and the portfolio of projects (Mun, 2016).

The IRM methodology is a systematic technique to determine the best possible projects to pursue based on the statistical likelihood of their success. Using historical knowledge of defense acquisition programs and IT systems in both the government and commercial realms could improve the budgeting and scheduling processes. Determining the likely range of outcomes through dynamic statistical modeling may improve the program's performance. By better understanding the risk associated with various components, a more appropriate schedule and budget could be developed. IRM may also help determine which real options should be included in acquisition contracts. A high-risk program may need more options, such as the options to abandon, delay, or expand, based on its actual performance. Finally, IRM could prove useful in portfolio management, helping decision makers determine which programs to initiate when viewing the portfolio of other programs in progress and used operationally.

All organizations depend heavily on project planning tools to forecast when various projects will complete. Completing projects within specified times and budgets is critical to facilitate smooth business operations. In our high-technology environment, many things can impact schedule. Technical capabilities can often fall short of expectations. Requirements are insufficient in many cases and need further definition. Tests can bring surprising results—good or bad. A whole host of other reasons can lead to schedule slips. On rare occasions, we may run into good fortune and the schedule can be accelerated. Project schedules are inherently uncertain, and change is normal. Therefore, we should expect changes and find the best way to deal with them. So why do projects always take longer than anticipated? One reason is inaccurate schedule estimating.

It is important to understand the IRM process and how the techniques involved are related in a risk analysis and risk management context. As previously noted, this framework comprises eight distinct phases of a successful and comprehensive risk analysis implementation, going from a qualitative management screening process to creating clear and concise reports for management. The process was developed by the author based on previous successful implementations of risk analysis, forecasting, real options, valuation, and optimization projects both in the consulting arena and in industry-specific problems. These phases can be performed either in isolation or together in sequence for a more robust integrated analysis.

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V. CASE STUDY: NPS ACQUISITION RESEARCH PROGRAM

The Department of Defense (DoD) Acquisition Workforce Development Fund (DAWDF) was created to provide "funding for the recruitment, training, and retention of DoD acquisition personnel." The purpose of the DAWDF is to "ensure the DoD acquisition workforce (AWF) has the capacity, in both personnel and skills, needed to properly perform its mission; provide appropriate oversight of contractor performance; and ensure that the Department receives the best value for the expenditure of public resources." Within this context, Naval Postgraduate School (NPS) graduate students have been collaborators in multiple research opportunities in the Acquisitions Research Program (ARP) and can now bring these analytical skills to the AWF.

The ARP at NPS should be seen as an R&D organization that generates innovations from research that may take years to bear fruit. It should also be recognized that typical R&D organizations yield a small number of major breakthrough products and services, and ARP research output should be viewed the same way. ARP research studies provide estimates of the future increases in returns on investment (ROI) of technologies to support core U.S. Navy processes such as shipbuilding and ship maintenance. It is important here to clarify the definition of ROI: ROI = (Revenue – Investment cost)/Investment cost. Many DoD leaders see ROI as a measure of cost savings, often without reference to value added by an asset, intellectual capital, or other forms of value production. In a nonprofit or governmental organization, an ROI ratio requires a revenue surrogate (in common units, and establishing such units is what the knowledge value added, or KVA, methodology does). In the following summaries of the ROI on ship maintenance and shipbuilding core processes, the Housel and Mun ARP studies used Market Comps to establish an estimate of the price per common unit of output of core processes to provide a monetized revenue surrogate. The cost of doing this kind of research, performed by SMEs and professionals at NPS compared to the cost of doing such studies by a comparable consulting company (e.g., McKinsey) would likely be at least three times as much due to the steep learning curve by non-SMEs.

ARP research is focused on possible scenarios that might *add value*, *reduce cost*, *provide savings*, *add capabilities*, and *provide value-added insights* that will make acquisition processes more productive and efficient. In almost all cases, the research provides value forecasts that often take years to bear fruit (as was found when the Navy finally decided to use product lifecycle management [PLM], three-dimensional printing [3DP], and three-dimensional laser scanning technology [3D LST] in shipbuilding and maintenance after many ARP studies recommended doing so). This time lag is typical in almost all R&D efforts in commercial companies, and the influence of ARP studies on Navy acquisition practices is similarly constrained. With that understanding, we will endeavor to compute the ROI for the entire ARP program.

Return on Investment (ROI) Calculations Methodology

Using valuation best practices in industry, we perform ROI analysis on the ARP program from various points of view to triangulate the final ROI:

- Some research provides significant ROI if the processes, recommendations, and actionable intelligence are executed. The ARP research will take minimal credit for the potential ROI (i.e., 1/1000 of the ROI savings) and attribute it to the ARP research.
- We look at the worst-case scenario, where even if the research results are not implemented, there are still cost savings. This approach will generate the absolute minimal baseline of what the ARP ROI should be.
- In addition, graduate students (MS, MBA, PhD candidates) participate in the research, as well as attend symposiums. There is value in the knowledge and experience gained, and we will capture these intangibles using Knowledge Value Added methodologies to monetize and determine the knowledge-based ROI.
- Intangible and intrinsic value exists above and beyond any standard ROI calculations. These include the interactions of sponsors with researchers, graduate students, and faculty, and program executive offices and commands with researchers; the live interactions of participants at the annual symposiums; and the knowledge dissemination.

1. Research-based ROI

Naval research and education are not separate tasks but tend to coexist alongside the innovation engines of the country. Several ARP studies provided estimates of the potential ROI increases in Navy ship maintenance and shipbuilding core processes. The following tables summarize the results of the ship maintenance and shipbuilding ROI increase estimates from incorporating three technologies into core processes. Table 1 shows that the detailed design and outfitting phases of shipbuilding benefit the most from use of the technologies, and that the sea trials and post-shakedown maintenance benefit the least. The ROI increases by 329% for that particular research, with an estimated potential savings for one ship of $296.91 Million. This represents a savings of 24.74% ($296.91 Million ÷ $1,200 Million) of the total cost to the Navy. We selected this research as an illustration of potential savings assuming the recommendations are carried out by the Navy. Therefore, these savings are estimated to be an average of $2.70 Billion per year ($296.91 Million per Ship × 264 Ships ÷ 29 Years). The cost of this singular research was approximately $120,000, which yields a research ROI of 240,000% even if only a single ship implements the methodology. Even with several orders of magnitude off, the ROI would still yield a highly significant percentage. We would maintain that the ARP research's contribution to this specific project alone, even with a highly conservative estimate that it is worth 1/1000 of ROI, is above **240%.**

Table 2 shows research on a Make or Buy analysis of the impacts of whether the Navy should execute 3D printing operations, 3D laser scanning technology, and collaborative product lifecycle management on ship maintenance and modernization cost savings that had ROI of the common unit of output (high-, medium-, or low-complexity parts). They range from **103%** to **1120%** in ROI per year per ship, averaging at 600%. These ROI values can be multiplied by a factor of 100 over the next 10 years when more ships implement the recommendations. Again, we would maintain that the ARP research's contribution to this specific project alone, even with a highly conservative estimate that it is worth 1/1000 of ROI, is above **600%**. .

Table 1 – ROI Projections for Shipbuilding Using PLM, 3DP, and 3D LST Technologies

No.	PROCESS / PHASE	As-is ROI	To-be ROI	Change in ROI	Automation Tools
1	**Concept design**	-2%	94%	**96%**	AM, PLM
2	**Detailed design**	561%	1826%	**1265%**	AM, PLM
3	**Preconstruction planning**	218%	244%	**25%**	PLM
4	**Block fabrication**	-67%	-31%	**36%**	3DLS, AM, PLM
5	**Block assemby and outfitting**	-17%	116%	**133%**	3DLS, AM, PLM
6	**Keel laying and block erection**	-63%	1%	**64%**	3DLS, AM, PLM
7	**PreDelivery outfitting**	505%	1270%	**764%**	3DLS, AM, PLM
8	**System testing**	280%	582%	**301%**	3DLS, PLM
9	**Sea trials**	1018%	961%	**-57%**	PLM
10	**PostDelivery outfitting**	476%	1243%	**767%**	3DLS, AM, PLM
11	**PostDelivery tests**	239%	282%	**42%**	PLM
12	**PostShakedown maintenance**	221%	201%	**-20%**	PLM
	TOTALS	**135%**	**464%**	**329%**	

Table 2 – ROI Projections for Shipbuilding by Part Complexity

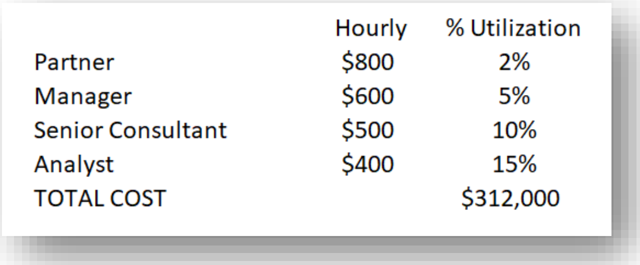
Part Complexity (% of total parts)		High (25%)		Medium (50%)		Low (25%)	
	Part Manufacturer	Industry	Navy	Industry	Navy	Industry	Navy
% Made by Navy	0	573%	NA	151%	NA	12%	NA
	25	NA	1120%	151%	NA	12%	NA
	50	NA	1120%	236%	510%	12%	NA
	75	NA	1120%	NA	358%	12%	NA
	100	NA	1120%	NA	358%	NA	103%

It might be argued that the Navy was going to incorporate these technologies years ago. However, there is scant evidence that the Navy leadership planned to do so prior to or during the years that the ARP studies were completed and delivered to PEO Ships leaders. Even if these studies made minor contributions to moving the Navy in the direction of adopting these technologies in ship maintenance and shipbuilding, the potential cost savings and ROI increases would more than justify the investments in this ARP research, even with our highly conservative assumption of contributing 1/1000 to the overall ROI. If the Navy had moved to incorporate these cost saving measures within four years (to allow for the typical learning curve of moving to incorporate technologies within core processes) of the original ARP studies, the result would have been many years of cost savings. Even if the ARP study cost savings estimates were off by an order of magnitude, they would well have been large enough to justify the overall investments in the ARP research studies.

2. Worst-Case Scenario ROI

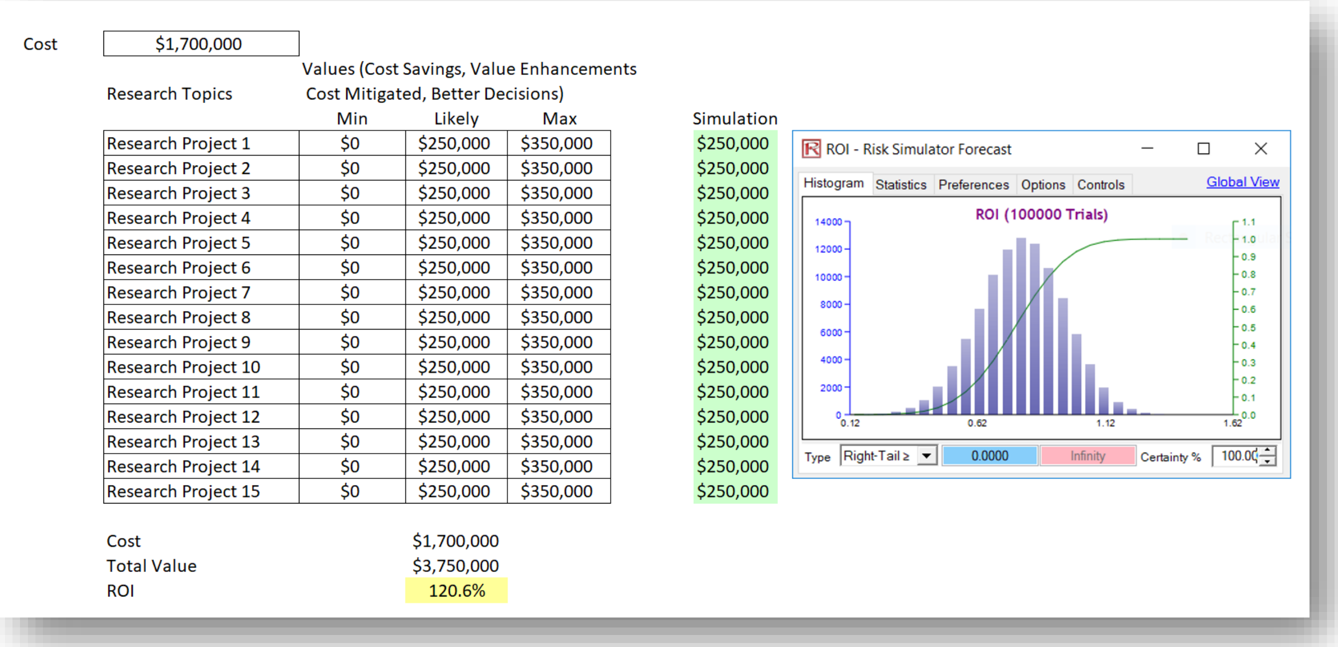
Next, we can show the absolute worst-case scenario ROI in Table 3 and Figure 9. The annual ARP cost is \$1.7M with 15 research projects on average. If done similarly by a third-party consulting company such as PricewaterhouseCoopers or McKinsey, the research usually runs around \$250,000–\$350,000 over the course of one year. For instance, the standard research takes 12 months (250 days × 8 hours per day = 2,000 hours per year). In addition, a standard project requires a partner, manager, and, at the very least, a senior consultant and analyst. Their rates are shown below. Even with the assumption that only 2% to 15% of their hours are used for the project, the average cost is \$312,000 per research project. Table 3 illustrates the computations.

Table 3 – A Standard PwC or McKinsey Research Program Cost



	Hourly	% Utilization
Partner	\$800	2%
Manager	\$600	5%
Senior Consultant	\$500	10%
Analyst	\$400	15%
TOTAL COST		\$312,000

Using the average cost computed, we set a range of \$250,000 most likely to \$350,000 maximum value for each research program as shown in Figure 9. To incorporate a worst-case scenario, we set the minimum value as \$0 (i.e., the research results are never operationalized, and the recommendations are never executed). A risk-based Monte Carlo simulation of 100,000 trials was run, and we see from Figure 9 that the average ROI, even on the worst-case scenario, is **120.6%**, with a 100% probability that the ROI of the ARP program returns a positive value. In other words, assuming that we separate and put aside for the moment the actual and significant value of the actionable intelligence from the research programs, and focus solely on the cost savings of the research alone, we generate a value of \$3.75M for the investment of \$1.7M for research and operating expenses, creating an ROI of **120.6%** in this worst-case scenario.



Cost $1,700,000

Research Topics	Values (Cost Savings, Value Enhancements Cost Mitigated, Better Decisions) Min	Likely	Max	Simulation
Research Project 1	$0	$250,000	$350,000	$250,000
Research Project 2	$0	$250,000	$350,000	$250,000
Research Project 3	$0	$250,000	$350,000	$250,000
Research Project 4	$0	$250,000	$350,000	$250,000
Research Project 5	$0	$250,000	$350,000	$250,000
Research Project 6	$0	$250,000	$350,000	$250,000
Research Project 7	$0	$250,000	$350,000	$250,000
Research Project 8	$0	$250,000	$350,000	$250,000
Research Project 9	$0	$250,000	$350,000	$250,000
Research Project 10	$0	$250,000	$350,000	$250,000
Research Project 11	$0	$250,000	$350,000	$250,000
Research Project 12	$0	$250,000	$350,000	$250,000
Research Project 13	$0	$250,000	$350,000	$250,000
Research Project 14	$0	$250,000	$350,000	$250,000
Research Project 15	$0	$250,000	$350,000	$250,000

Cost	$1,700,000
Total Value	$3,750,000
ROI	120.6%

Type Right-Tail ≥ 0.0000 Infinity Certainty % 100.00

Figure 9: ROI Projections Worst-Case Scenario

3. Value of Knowledge and ROI

The ARP researchers use graduate students to assist in their work. These MS, MBA, and PhD students are active duty Navy and Marine officers who will return to their commands armed with the valuable hands-on practical research knowledge and experience that are second to none. We quantify these knowledge value-added learnings from the ARP research they have conducted and monetize using the KVA approach as seen in Table 4. The ROI on a single ARP research program is calculated to be 253%.

The calculations assume that the thesis students will populate the future AWF. We also assume that the acquisition case studies that have been developed from ARP research and are used to teach a wide variety of acquisition, business, public policy, and information science classes provide important learnings that translate into future acquisitions workforce knowledge. We have normalized the knowledge into common units of learning time and assumed that the graduate students will apply their obtained acquisitions knowledge to acquisition challenges and opportunities for adding value to the core acquisition processes. Those students who attend the Acquisition Symposium are also likely to pick up some valuable key learnings that they can then apply to future acquisition decision-making situations. These opportunities to obtain acquisition knowledge in these three learning contexts are summarized in Table 4. These estimates are very conservative and represent one year of learning opportunities. The central assumption in this analysis, as in most educational value analyses, is that the students will translate their acquisition learnings into knowledge that will be put to use once they leave NPS. As it is put to use, it will generate value for the acquisition workforce.

Table 4 – Value of Knowledge

	EXPERIENTIAL GRADUATE ARP RESEARCH	GRADUATE STUDENT (ARP CASES)	ARP SYMPOSIUM PARTICIPANT LEARNER
Number of Days per Year	200	200	200
Normalized Total Knowledge Units	100	100	100
Accumulated Knowledge Used	10%	5%	1%
Hours/Day Used	4	4	4
Units of Knowledge Used/Hour	10	5	1
Total Knowledge Units	8000	4000	800
Consultant Annual Salary	$150,000	$150,000	$150,000
Comp Price Per Unit of Knowledge	$18.75	$18.75	$18.75
Daily Value	$750	$375	$75
Value/Year Per Student	$150,000	$75,000	$15,000
Average Students Exposed	10	50	50
Valuation for Each Category	$1,500,000	$3,750,000	$750,000
Total Valuation of Knowledge	$6,000,000	$6,000,000	
ARP Total Cost (Research Only)	$1,700,000		
ROI of ARP in Knowledge Terms	252.94%		

In summary, we can quantify that the ARP's ROI based on an annual investment of $1.7M will range from the absolute worst case of 121% to an average of 240%–600% for each specific program (Table 5). The KVA method pegs the ROI at 253%. Therefore, using standard industry best practices, we conclude the average conservative ROI for the entire ARP program to be approximately 304% for the annual $1.7M total investment for research and operating expenses. These ROI estimates should be seen as the minimal value because significant intangible value exists when we run research programs with uniformed graduate students and when we hold the annual symposiums.

Table 5 – Summary ROI for Military Research and Development

ROI for Military Research and Development (e.g., ARP)	
Minimal Worst-Case ROI	121.00%
Most Likely ROI	304.00%
Range of ROI Depending on Program	240% – 600%

4. Intangible Value

NPS graduates approximately 1,200 students annually, with a competitive advantage in technical education and applied research, that is both applicable and responsive to the needs of DON. This agility can be seen in its many research programs, such as the ARP discussed above, with its key research results presented in its annual symposium. Hundreds of research projects and programs are being churned out at NPS annually, and the ARP is only an example of the large swatch of knowledge generated.

The symposium presentations include the interactions of sponsors with researchers, graduate student and faculty, and program executive offices and commands with researchers; the live interactions of participants at the annual symposiums; and the knowledge dissemination. All of this hands-on experiential research and networking helps prepare NPS graduate students to be the next wave of a highly qualified AWF.

VI. CASE STUDY: THE VALUE AND ROI OF NPS

Strategic and Intangible Value

In this section, we look at the Naval Postgraduate School (NPS) located in Monterey, California, as an analytical case study of return on investment (ROI) on military education. One could easily agree that NPS is the U.S. Navy's postgraduate university as it is essential to the Navy's education continuum to ensure the combat-effectiveness of both military and civilians. NPS is integral to joint and combined professional military education, making it a critical element in our country's national security strategy and priority.

In fact, NPS is at the forefront of providing specialized graduate, postgraduate, and certificate-level programs supporting U.S. national security policies and priorities, including counterterrorism, homeland security, and security cooperation. And the "synergistic combination of graduate education in disciplines and curricula critical to the future of our defense establishment with high-impact research in crucial technologies directly relevant to DoD's mission is simply not found in either national laboratories with no capacity or interest in educating military officers or in civilian universities that engage in little or no defense R&D" (Naval Postgraduate School, 2012a).

NPS graduates continue on their military and civilian careers with distinction. As an illustration, Appendix I shows a list of the latest distinguished NPS alumni and hall of fame NPS alumni. These include prominent flag officers, general officers, naval captains, assistant secretaries, and deputy secretaries of defense, as well as other foreign dignitaries. This is only a partial list but makes the case for the value of an NPS education to the government.

The quality of NPS's education is comparable, if not superior, to many civilian private universities. NPS faculty members have won numerous prestigious awards (e.g., 2014 INFORMS Koopman Prize, J. Steinhardt Prize, MORS Barchi Prize, Hugh G. Nott Prize, and so forth), while the school is accredited by national educational organizations (e.g., WACS, ABET, AACSB, NASPAA), and generates multiple patents, research publications, and print publications every year.

While the Navy has the option to send its officers to private and public universities, an analysis of alternatives shows that in doing so, the Navy would sacrifice its agility and responsiveness, and potentially even incur a higher cost. In fact, according to the *NPS Value Book,* "cost comparisons are being made erroneously between civilian universities market price (tuition) and NPS full costs. Tuition covers 15–25% of public and 25–30% of private universities' full cost … Analysis has shown NPS to be average to below average in total costs" (Naval Postgraduate School, 2012). The N81-led study group also concluded that NPS was "$22M cheaper than CIVINS would be for providing a fully comparable education. A 1993 analysis by NPS using N81 study data showed that NPS had a cost per class hour of $135 versus $176 for CIVINS" (N09BC, 1996). In addition to the higher cost of external civilian universities, the lack of direct applications to the military and the loss of control by the Secretary of Navy or Secretary of Defense over how robust and rigorous each curriculum should be imply that the expectation that civilian universities can meet the Navy's needs over time in terms of military education is false.

Officers are sent to NPS when the educational skills involve DoD-specific knowledge that is not readily available at civilian universities. ADM Henry H. Mauz, Jr. (Retired) and William Gates illustrated that NPS is a good return on investment and called studies to find alternative means of providing graduate education at less cost "flawed by imbalanced analysis, inadequate research, and preordained outcomes." They compared NPS to civilian universities and showed how NPS is far superior. They stressed the ability of NPS to "quickly adapt curricula to changing needs of all military services, and that NPS is unique in offering naval and defense curricula" (Mauz and Gates, 2000). In their report, the authors stressed the following items:

- NPS provides militarily relevant studies that meet Navy and Marine Corps subspecialty and general education requirements.
- NPS curricula are subject to biennial Navy-flag-level sponsor review and updates for military relevancy, and new courses and programs can be updated quickly.
- Entrance to NPS is controlled by military performance and demonstrated aptitude instead of relying on graduate-level standardized tests and undergraduate grade-point averages.
- NPS provides able and motivated officers the opportunity to transition from one undergraduate area to a different graduate major.
- NPS provides refresher courses to allow students to renew academic skills after several years of on-the-job performance.
- Faculty and students participate in more than 500 research projects per year on issues of interest to sponsoring or funding agencies from the Department of the Navy and throughout the U.S. government.

- The NPS student body combines junior officers from the Navy, Marine Corps, Army, Air Force, National Guard, defense agencies, and more than 60 foreign countries to explore technical, operational, and strategic problems.

Cost effectiveness of an NPS education was previously reported in the Memorandum for the Deputy Chief of Naval Operations (N81/3U639949, 1993). Specifically, it stated that if NPS and civilian programs are of different duration (e.g., 18 versus 28 months), any cost comparison must include the students' salaries and benefits. The "Department of the Navy's Director, Assessment Division, estimated that the annual cost of salary, benefits, and housing per NPS-resident officer totaled $63,300, compared to $72,300 per officer student at civilian institutions. The higher civilian cost reflects the fact that most NPS officers live in base housing" (N81/3U639949, 1993).

NPS was "rated high by the BRAC Technical Joint Cross Service Group (TJCSG) when they examined 146 technical facilities regarding their value to defense RDT&E" (Military Value Analysis, 2005). The report identified the most important 13 technical areas in developing military strength, then evaluated each technical facility over three functional areas: research, development and acquisition, and test and evaluation.

Rather than only considering the ROI of NPS, we should also be focusing on how to better maximize the return on our investment.

> A comparison of the costs associated with a degree earned from the Naval Postgraduate School and a similar degree earned from a comparable civilian university was performed. Although the degrees may be the same when displayed on a sheepskin, and surely just as challenging in their pursuit, a civilian course of study almost certainly does not represent the same tailored, defense-centric, militarily career-enhancing curriculum provided by NPS. This is a crucial flaw inherent in any cost comparison. Because, in fact, curricular requirements at NPS include Educational Skill Requirements (ESRs) dictated by the Secretary of the Navy that are intended to broaden the military student's educational experience. For instance, NPS provides JPME coursework on campus from dedicated War College faculty, so that officers can satisfy both their masters and joint military requirements during a single tour. Additional coursework is also required to ensure the student appreciates the military relevance of the academic subject material, thereby enabling immediate application upon rejoining the operational force. Hence, additional credit hours of instruction are built into NPS curricula to meet ESRs. Similar courses are not available at civilian universities and represent a hidden, but necessary, cost in NPS' budget. (U.S. Naval Institute, 2000)

Naval maritime supremacy requires a Navy-oriented focus to meet the technical and professional challenges of the 21st century and beyond. "The Navy is in the higher education business because of the required focus on naval professional development, meeting the requirements of technological innovation, ability to exercise quality control, as well as optimizing Navy colleges capabilities" (N09BC, 1995). For the Navy, undergraduate, graduate, and professional military education is an investment, and, like any investment, its returns need to be evaluated.

Tactical and Tangible Quantitative ROI for NPS

In order to quantitatively measure a robust ROI for NPS educational programs, the quantifiable benefits and costs are first obtained and analyzed, and later invoked in a lifecycle cost-benefit model with simulation. ROI is mainly a monetary or economic metric. This means we can only determine ROI based on the main tangible monetary benefits of an NPS education, such as the lower tuition costs as well as the higher retention rate of NPS graduates. The retention rates modeling uses the Analytical Framework approach whereas the lifecycle cost-benefit modeling use the Work Lifecycle approach previously examined. A modification of the Systems Utilization approach and Frequency Quantity approach is used in the lifecycle model, complemented with Integrated Risk Management methods in applying Monte Carlo simulation. The following subsections break down the methods into quantized analytical chunks.

NPS Graduates Show Higher Retention Rates in the Navy

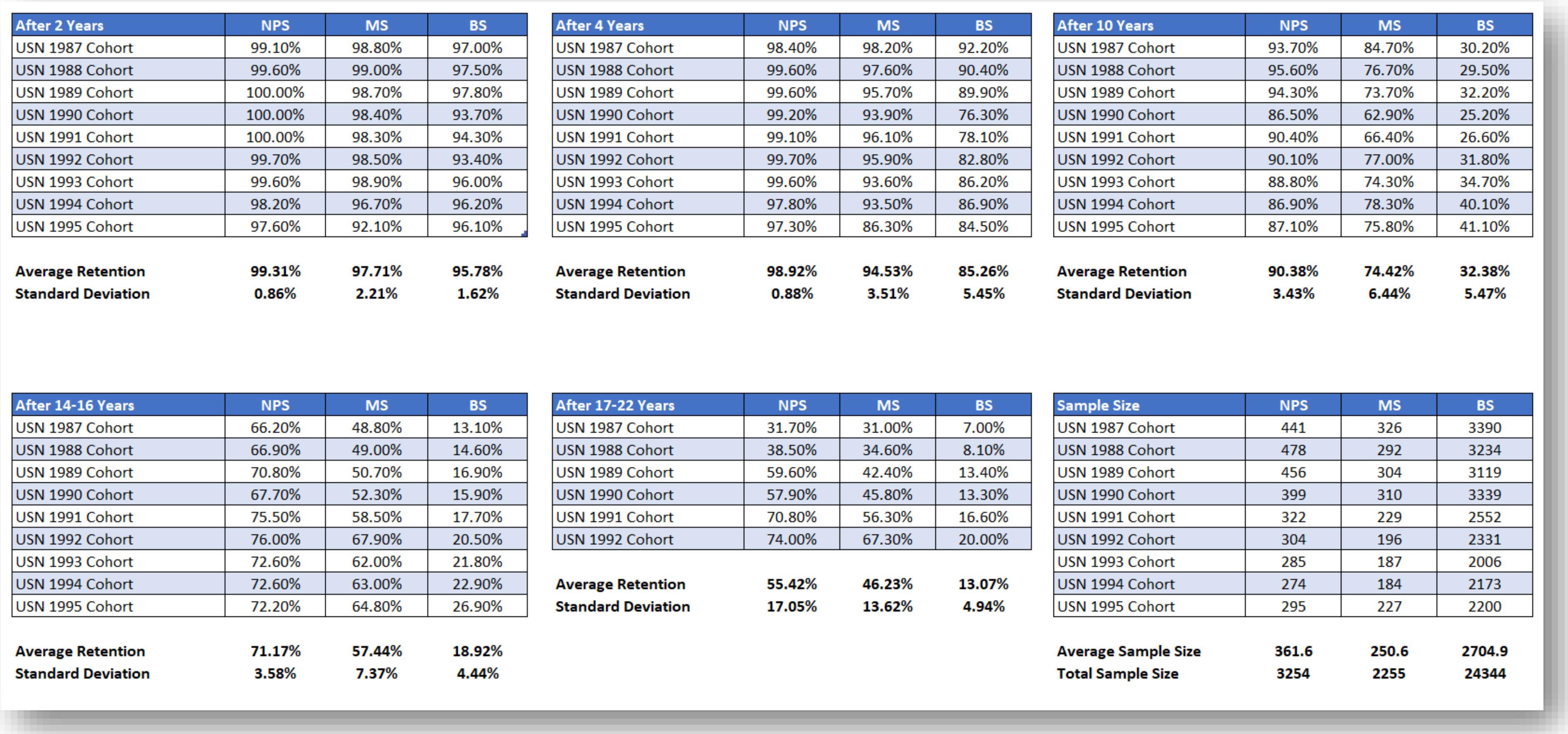
Figure 10 shows the DoD retention rates of NPS graduates (both at the MS and PhD levels), non-NPS civilian MS-level (masters or equivalent degree) graduates, and non-NPS civilian BS-level (bachelor's or equivalent undergraduate degree) graduates. These non-NPS graduates can come from a variety of nonmilitary private and public universities. Cohort data from the 1987 through 1995 graduating classes were obtained (Naval Postgraduate School, 2012b [Office of Institutional Research]). For instance, we see that 2 years after graduation, the retention rates are relatively high for all three groups, ranging from 99.31% to 95.78% on average. This high rate of retention the first few years is to be expected as officers sent to graduate programs typically are required to "pay back" their education costs with guaranteed service for several years. In comparison, after a span of 17–22 years post-graduation, the NPS graduates showed a 55.42% DoD retention compared to 46.23% for non-NPS MS graduate programs and 13.07% for other non-NPS BS undergraduate programs. The total sample sizes for the data aggregation were 3,254 for NPS, 2,255 from other graduate programs, and 24,344 from other undergraduate programs.

Figures 11 to 13 illustrate the cross-sectional retention bands for the three groups. All three charts indicate a smooth laminar flow across all cohorts with respect to 2- and 4-year retention rates. There seems to be more disturbance around the 10-year milestone, especially for the undergraduate degree holders, and less so for the NPS graduates. The highest volatility can be seen in the undergraduate degree holders' cross section starting from the 10-year through the 15-year and 20-year milestones.

There is a sharp 10-year decline in retention, as can be seen in the time-series line chart in Figure 14. The drop is most precipitous for the undergraduate degree holders. The vertical distances between these lines indicate the differences in retention rates. There is a visibly significant difference between the BS and MS/NPS graduates, and a smaller but visually distinct difference between non-NPS MS and NPS graduates.

The average rates across these various cohorts in time are shown in Figure 15. There are certainly differences among all groups, and these differences are tested statistically using a parametric analysis of variance for single factor multiple treatments (ANOVA) and confirmed with a nonparametric Kruskal-Wallis (KW) test. The null hypotheses tested were that, for each retention milestone, there is no statistically significant difference among all three groups of graduates when comparing all groups at once. While both the ANOVA and KW tests can identify whether there are any differences among the three groups tested, they do not identify where the differences come from. Hence, further analyses using the one-tailed paired parametric t-test of two independent variables with unequal variances were run on every combination of the three groups, and the results were confirmed using the nonparametric two-variable Wilcoxon signed rank test. The parametric tests were applied as we have large sample sizes as discussed previously, for example, up to 24,344 graduates in all the cohorts for the non-NPS undergraduate programs. This allows us to take advantage of the law of large numbers and the central limit theorem, justifying the use of parametric tests. The nonparametric tests were also applied because the averages were used and the larger sample sizes have been reduced to a smaller subset, where the underlying normality assumption may or may not be violated. In addition, the natural truncation of percentages (i.e., 0% to 100%) calls for the use of nonparametric methods.

Figure 16 shows the results of these tests. With an alpha significance level set at α = 0.05, the one-tailed directional tests (the null hypothesis tested was that there is no difference in retention rates, versus the alternative hypothesis that the NPS graduates had higher retention rates than the non-NPS graduate degree holders, and greater than the non-NPS undergraduate degree holders). We see that in almost all cases, NPS graduates have statistically significantly (denoted by with an asterisk *) higher retention rates than all non-NPS graduates. The only area showing non-significance is the 20-year average retention rates between NPS graduates and non-NPS graduate degree holders. This might be due to the authorized strength limitations imposed by Congress on the number of flag and general officers (Title 10 U.S. Code § 526: Authorized strength: general and flag officers on active duty), but further investigation is warranted in future research.



After 2 Years	NPS	MS	BS
USN 1987 Cohort	99.10%	98.80%	97.00%
USN 1988 Cohort	99.60%	99.00%	97.50%
USN 1989 Cohort	100.00%	98.70%	97.80%
USN 1990 Cohort	100.00%	98.40%	93.70%
USN 1991 Cohort	100.00%	98.30%	94.30%
USN 1992 Cohort	99.70%	98.50%	93.40%
USN 1993 Cohort	99.60%	98.90%	96.00%
USN 1994 Cohort	98.20%	96.70%	96.20%
USN 1995 Cohort	97.60%	92.10%	96.10%
Average Retention	**99.31%**	**97.71%**	**95.78%**
Standard Deviation	**0.86%**	**2.21%**	**1.62%**

After 4 Years	NPS	MS	BS
USN 1987 Cohort	98.40%	98.20%	92.20%
USN 1988 Cohort	99.60%	97.60%	90.40%
USN 1989 Cohort	99.60%	95.70%	89.90%
USN 1990 Cohort	99.20%	93.90%	76.30%
USN 1991 Cohort	99.10%	96.10%	78.10%
USN 1992 Cohort	99.70%	95.90%	82.80%
USN 1993 Cohort	99.60%	93.60%	86.20%
USN 1994 Cohort	97.80%	93.50%	86.90%
USN 1995 Cohort	97.30%	86.30%	84.50%
Average Retention	**98.92%**	**94.53%**	**85.26%**
Standard Deviation	**0.88%**	**3.51%**	**5.45%**

After 10 Years	NPS	MS	BS
USN 1987 Cohort	93.70%	84.70%	30.20%
USN 1988 Cohort	95.60%	76.70%	29.50%
USN 1989 Cohort	94.30%	73.70%	32.20%
USN 1990 Cohort	86.50%	62.90%	25.20%
USN 1991 Cohort	90.40%	66.40%	26.60%
USN 1992 Cohort	90.10%	77.00%	31.80%
USN 1993 Cohort	88.80%	74.30%	34.70%
USN 1994 Cohort	86.90%	78.30%	40.10%
USN 1995 Cohort	87.10%	75.80%	41.10%
Average Retention	**90.38%**	**74.42%**	**32.38%**
Standard Deviation	**3.43%**	**6.44%**	**5.47%**

After 14-16 Years	NPS	MS	BS
USN 1987 Cohort	66.20%	48.80%	13.10%
USN 1988 Cohort	66.90%	49.00%	14.60%
USN 1989 Cohort	70.80%	50.70%	16.90%
USN 1990 Cohort	67.70%	52.30%	15.90%
USN 1991 Cohort	75.50%	58.50%	17.70%
USN 1992 Cohort	76.00%	67.90%	20.50%
USN 1993 Cohort	72.60%	62.00%	21.80%
USN 1994 Cohort	72.60%	63.00%	22.90%
USN 1995 Cohort	72.20%	64.80%	26.90%
Average Retention	**71.17%**	**57.44%**	**18.92%**
Standard Deviation	**3.58%**	**7.37%**	**4.44%**

After 17-22 Years	NPS	MS	BS
USN 1987 Cohort	31.70%	31.00%	7.00%
USN 1988 Cohort	38.50%	34.60%	8.10%
USN 1989 Cohort	59.60%	42.40%	13.40%
USN 1990 Cohort	57.90%	45.80%	13.30%
USN 1991 Cohort	70.80%	56.30%	16.60%
USN 1992 Cohort	74.00%	67.30%	20.00%
Average Retention	**55.42%**	**46.23%**	**13.07%**
Standard Deviation	**17.05%**	**13.62%**	**4.94%**

Sample Size	NPS	MS	BS
USN 1987 Cohort	441	326	3390
USN 1988 Cohort	478	292	3234
USN 1989 Cohort	456	304	3119
USN 1990 Cohort	399	310	3339
USN 1991 Cohort	322	229	2552
USN 1992 Cohort	304	196	2331
USN 1993 Cohort	285	187	2006
USN 1994 Cohort	274	184	2173
USN 1995 Cohort	295	227	2200
Average Sample Size	**361.6**	**250.6**	**2704.9**
Total Sample Size	**3254**	**2255**	**24344**

Figure 10: Retention Rates

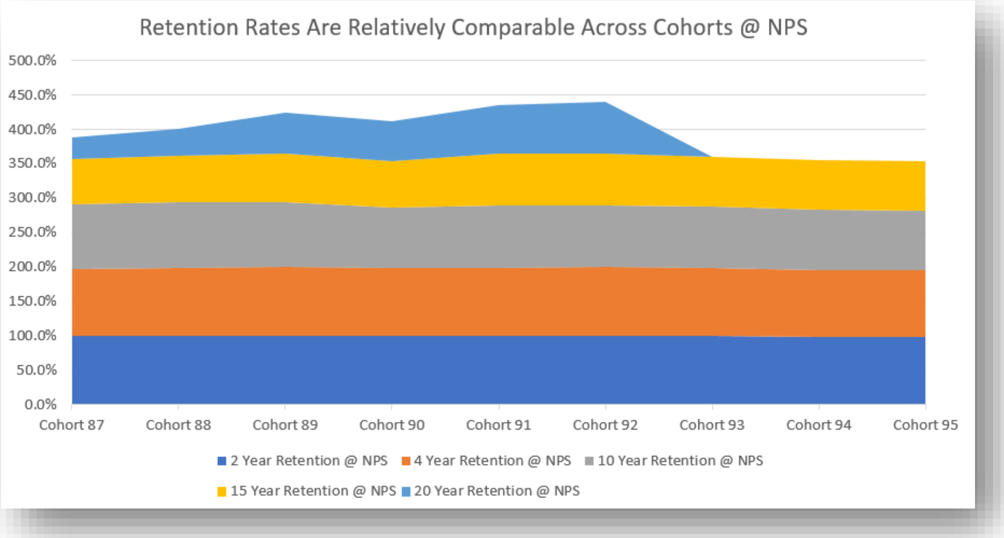


Figure 11: Cross-sectional Band of Retention Rates of NPS Graduates

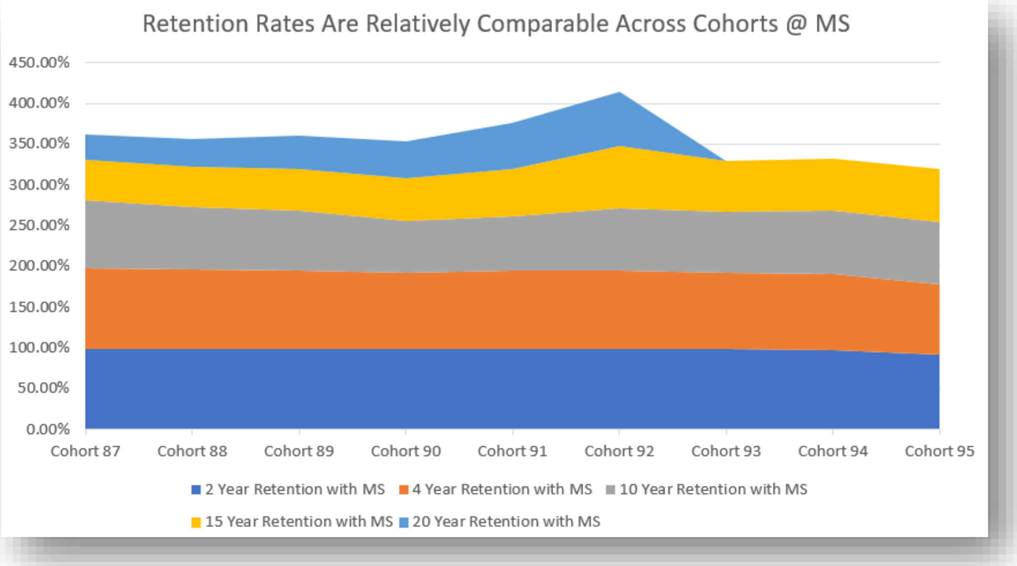


Figure 12: Cross-sectional Band of Retention Rates of Non-NPS Graduate Degrees

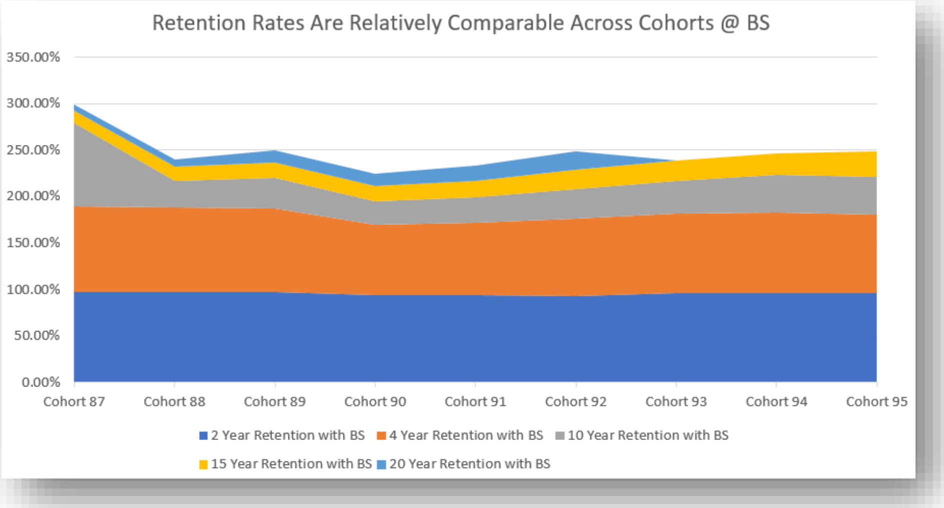


Figure 13: Cross-sectional Band of Retention Rates of Undergraduate Degrees

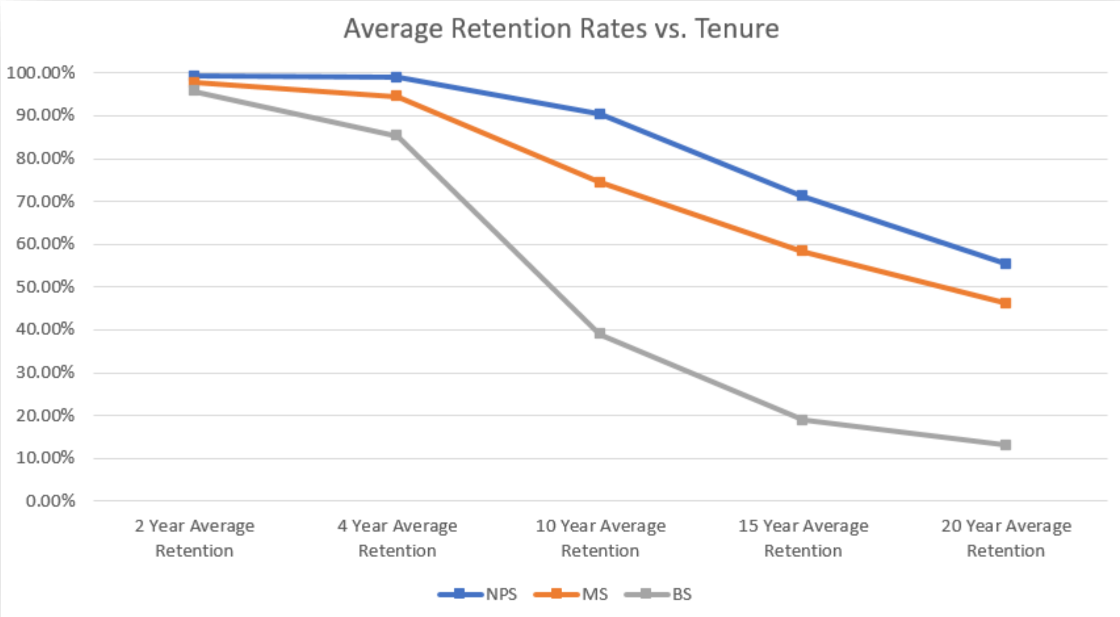
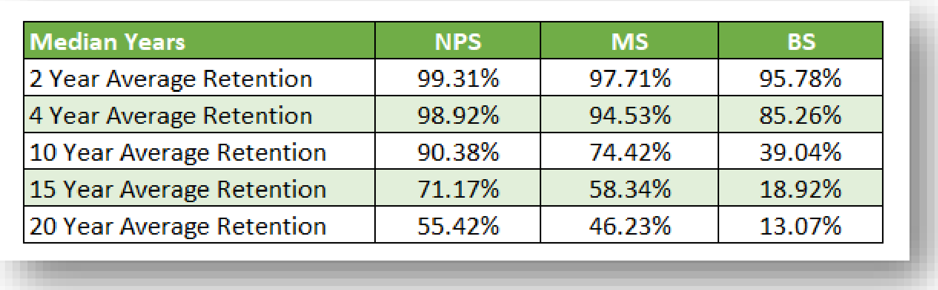
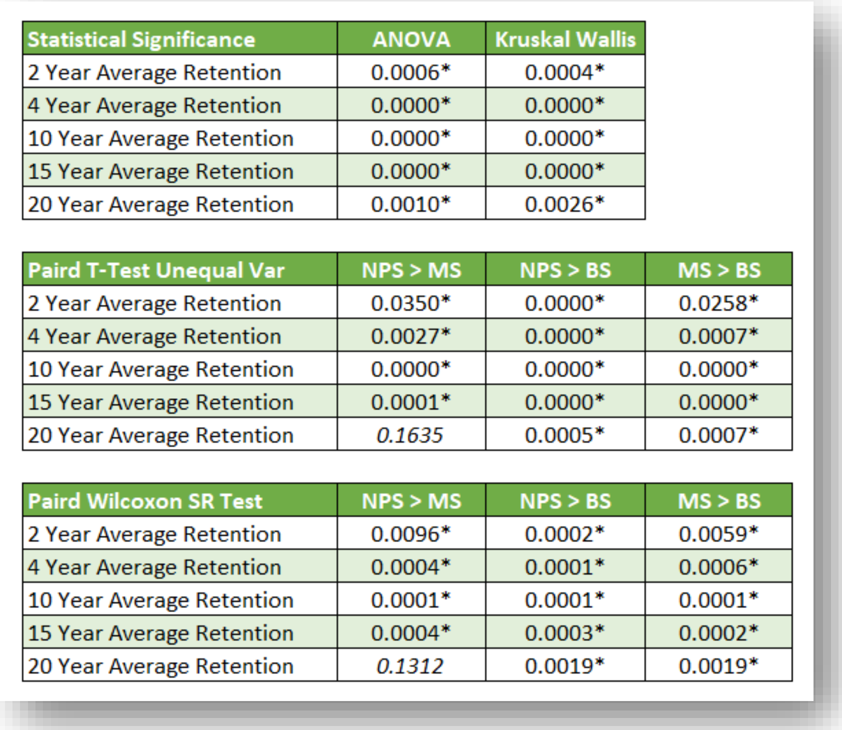


Figure 14: Time-series Line Charts of the Three Groups of Graduates



Median Years	NPS	MS	BS
2 Year Average Retention	99.31%	97.71%	95.78%
4 Year Average Retention	98.92%	94.53%	85.26%
10 Year Average Retention	90.38%	74.42%	39.04%
15 Year Average Retention	71.17%	58.34%	18.92%
20 Year Average Retention	55.42%	46.23%	13.07%

Figure 15: Average Retention Rates of All Cohorts for the Three Groups of Graduates



Statistical Significance	ANOVA	Kruskal Wallis
2 Year Average Retention	0.0006*	0.0004*
4 Year Average Retention	0.0000*	0.0000*
10 Year Average Retention	0.0000*	0.0000*
15 Year Average Retention	0.0000*	0.0000*
20 Year Average Retention	0.0010*	0.0026*

Paird T-Test Unequal Var	NPS > MS	NPS > BS	MS > BS
2 Year Average Retention	0.0350*	0.0000*	0.0258*
4 Year Average Retention	0.0027*	0.0000*	0.0007*
10 Year Average Retention	0.0000*	0.0000*	0.0000*
15 Year Average Retention	0.0001*	0.0000*	0.0000*
20 Year Average Retention	*0.1635*	0.0005*	0.0007*

Paird Wilcoxon SR Test	NPS > MS	NPS > BS	MS > BS
2 Year Average Retention	0.0096*	0.0002*	0.0059*
4 Year Average Retention	0.0004*	0.0001*	0.0006*
10 Year Average Retention	0.0001*	0.0001*	0.0001*
15 Year Average Retention	0.0004*	0.0003*	0.0002*
20 Year Average Retention	*0.1312*	0.0019*	0.0019*

Figure 16: ANOVA and Kruskal-Wallis Tests

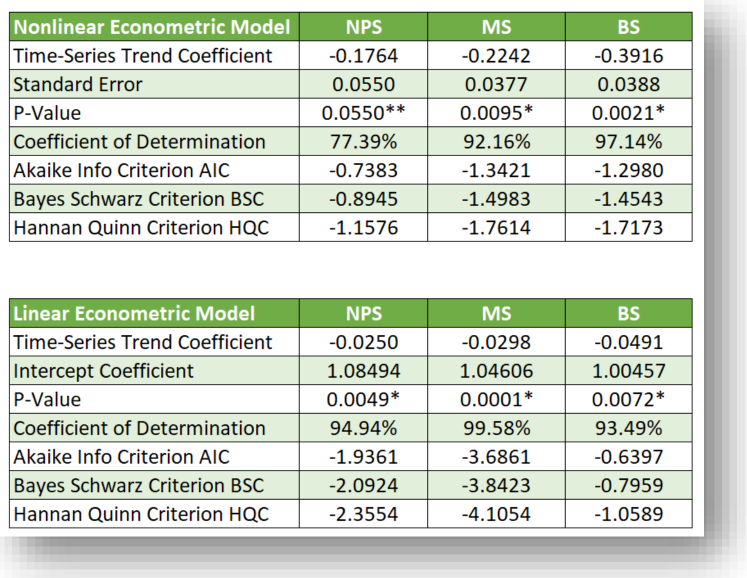
Retention Rates Are Fairly Predictable

Now that we have statistically established that NPS graduates tend to have a higher retention rate than non-NPS graduates, the question is whether this trend is predictable. Predictability is key for the DoD in terms of anticipating force readiness levels for the future. Having a more stable group of qualified Naval officers 10, 15, or 20 years out allows for the fleet to plan for future readiness and future capability levels.

A time-series indexed set of linear and nonlinear econometric models were tested, starting with simple linear and nonlinear functional forms (Figure 17). The coefficients of determination ranged from 77.4% to 99.6% predictive power, with adequate error measurements (Akaike, Bayes Schwarz, and Hannan Quinn criteria). Using these models, the retention rates were forecasted and compared against the actual rates and the forecast errors are shown in Figure 18. The mean absolute percentage error (MAPE) of predictions are computed and the median of these errors fluctuate between 0.01% and 3.34%, which corresponds to a median forecast error of between ±0.11% and ±4.42% as measured by the mean absolute deviation (MAD). The forecast results and actuals are also shown side by side in Figure 19 and visually in Figure 20.

Further modeling is required as although the initial error rates are well within reasonable bounds, we wish to see if more advanced functional forms can be used to predict these retention rates more accurately. Figures 21, 22, and 23 show examples of the more exhaustive econometric functional forms tested, including the standard linear and nonlinear models, followed by quadratic, loglinear, logistic, linear log, double log, reciprocal, and log reciprocal models.

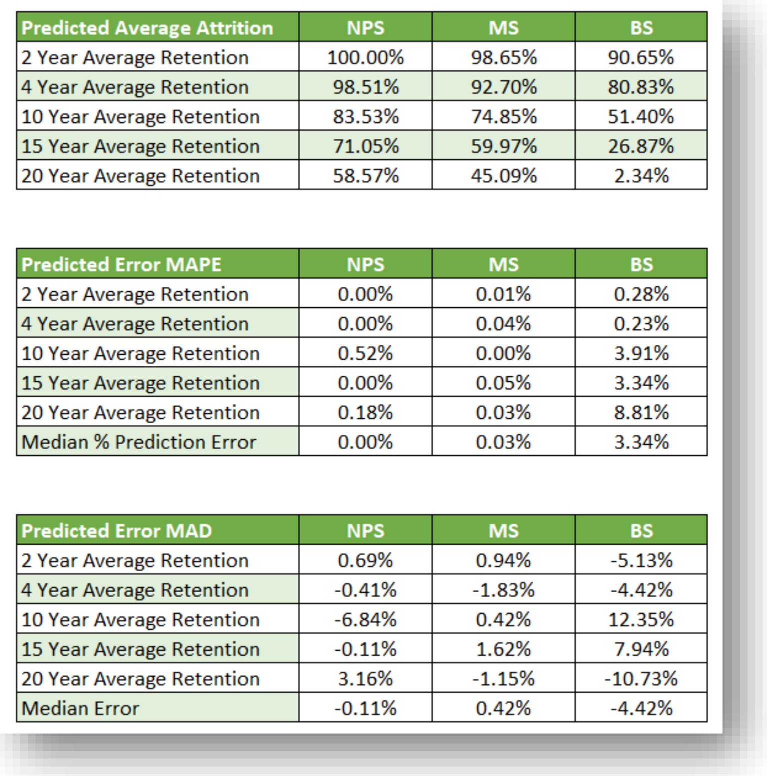
Using the best models for each group of graduates (denoted by double asterisks **), the retention rates were again modeled and compared against the actuals to determine their viability and prediction errors (Figure 24). We can see that using more complex functional forms provided higher efficacy levels and lower errors. Figure 25 shows the final prediction model against the actual rates. Using these best prediction models, we can now run a more comprehensive lifecycle cost model.



Nonlinear Econometric Model	NPS	MS	BS
Time-Series Trend Coefficient	-0.1764	-0.2242	-0.3916
Standard Error	0.0550	0.0377	0.0388
P-Value	0.0550**	0.0095*	0.0021*
Coefficient of Determination	77.39%	92.16%	97.14%
Akaike Info Criterion AIC	-0.7383	-1.3421	-1.2980
Bayes Schwarz Criterion BSC	-0.8945	-1.4983	-1.4543
Hannan Quinn Criterion HQC	-1.1576	-1.7614	-1.7173

Linear Econometric Model	NPS	MS	BS
Time-Series Trend Coefficient	-0.0250	-0.0298	-0.0491
Intercept Coefficient	1.08494	1.04606	1.00457
P-Value	0.0049*	0.0001*	0.0072*
Coefficient of Determination	94.94%	99.58%	93.49%
Akaike Info Criterion AIC	-1.9361	-3.6861	-0.6397
Bayes Schwarz Criterion BSC	-2.0924	-3.8423	-0.7959
Hannan Quinn Criterion HQC	-2.3554	-4.1054	-1.0589

Figure 17: Time-series Econometric Modeling Results

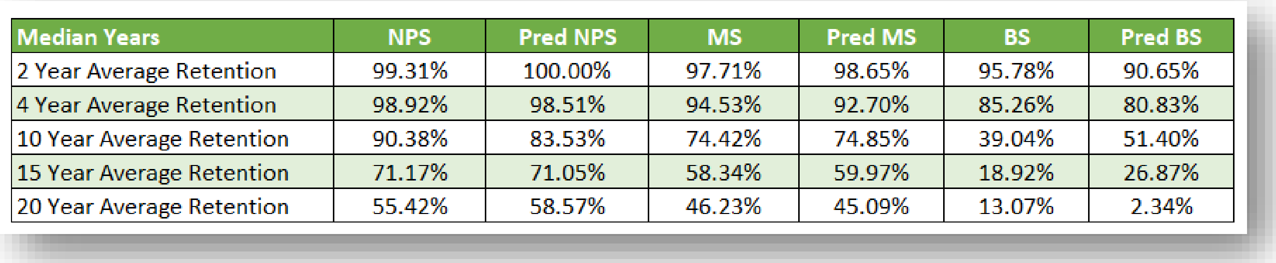


Predicted Average Attrition	NPS	MS	BS
2 Year Average Retention	100.00%	98.65%	90.65%
4 Year Average Retention	98.51%	92.70%	80.83%
10 Year Average Retention	83.53%	74.85%	51.40%
15 Year Average Retention	71.05%	59.97%	26.87%
20 Year Average Retention	58.57%	45.09%	2.34%

Predicted Error MAPE	NPS	MS	BS
2 Year Average Retention	0.00%	0.01%	0.28%
4 Year Average Retention	0.00%	0.04%	0.23%
10 Year Average Retention	0.52%	0.00%	3.91%
15 Year Average Retention	0.00%	0.05%	3.34%
20 Year Average Retention	0.18%	0.03%	8.81%
Median % Prediction Error	0.00%	0.03%	3.34%

Predicted Error MAD	NPS	MS	BS
2 Year Average Retention	0.69%	0.94%	-5.13%
4 Year Average Retention	-0.41%	-1.83%	-4.42%
10 Year Average Retention	-6.84%	0.42%	12.35%
15 Year Average Retention	-0.11%	1.62%	7.94%
20 Year Average Retention	3.16%	-1.15%	-10.73%
Median Error	-0.11%	0.42%	-4.42%

Figure 18: Forecast Errors of the Linear and Nonlinear Models



Median Years	NPS	Pred NPS	MS	Pred MS	BS	Pred BS
2 Year Average Retention	99.31%	100.00%	97.71%	98.65%	95.78%	90.65%
4 Year Average Retention	98.92%	98.51%	94.53%	92.70%	85.26%	80.83%
10 Year Average Retention	90.38%	83.53%	74.42%	74.85%	39.04%	51.40%
15 Year Average Retention	71.17%	71.05%	58.34%	59.97%	18.92%	26.87%
20 Year Average Retention	55.42%	58.57%	46.23%	45.09%	13.07%	2.34%

Figure 19: Actual vs. Predicted Retention Rates

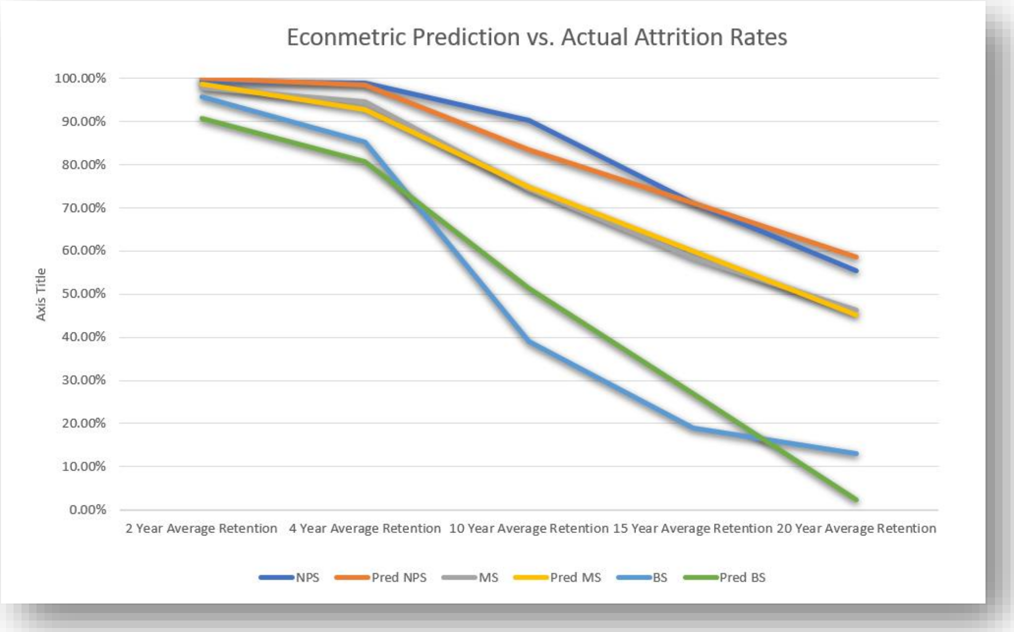
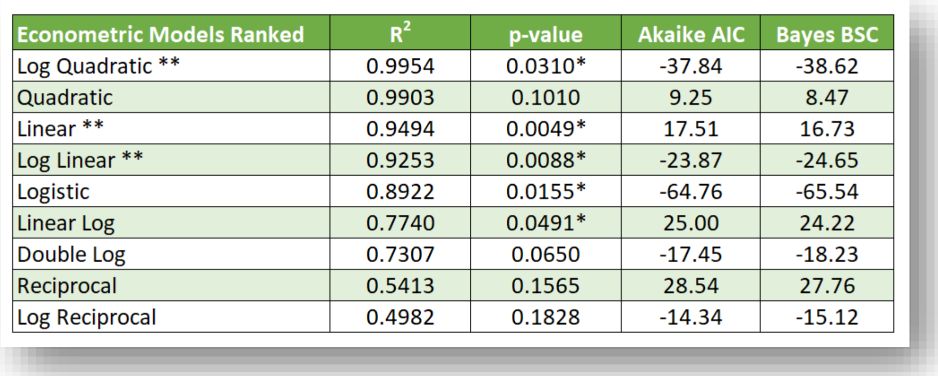
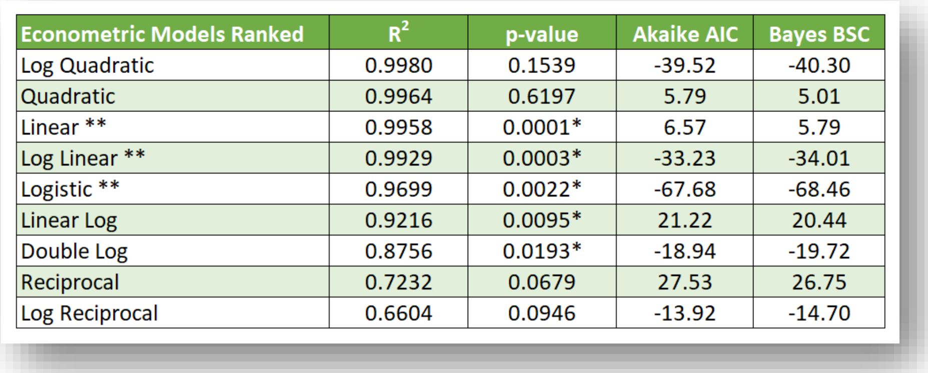


Figure 20: Visual Representation of the Goodness of Fit of the Forecasts



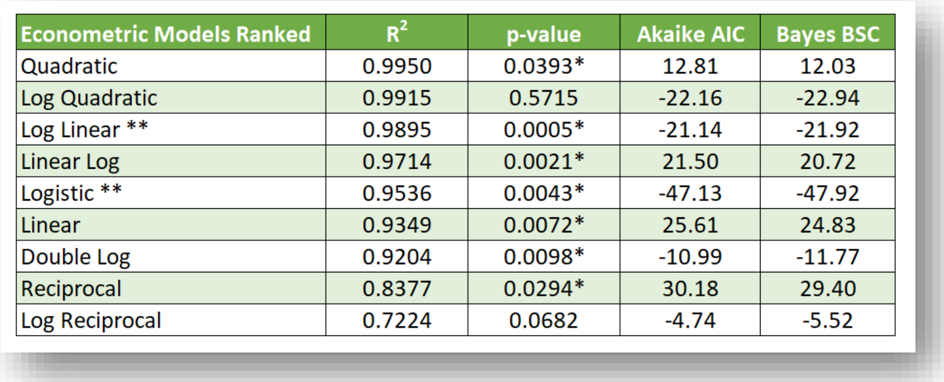
Econometric Models Ranked	R^2	p-value	Akaike AIC	Bayes BSC
Log Quadratic **	0.9954	0.0310*	-37.84	-38.62
Quadratic	0.9903	0.1010	9.25	8.47
Linear **	0.9494	0.0049*	17.51	16.73
Log Linear **	0.9253	0.0088*	-23.87	-24.65
Logistic	0.8922	0.0155*	-64.76	-65.54
Linear Log	0.7740	0.0491*	25.00	24.22
Double Log	0.7307	0.0650	-17.45	-18.23
Reciprocal	0.5413	0.1565	28.54	27.76
Log Reciprocal	0.4982	0.1828	-14.34	-15.12

Figure 21: NPS Graduates Attrition Rate Functional Forms



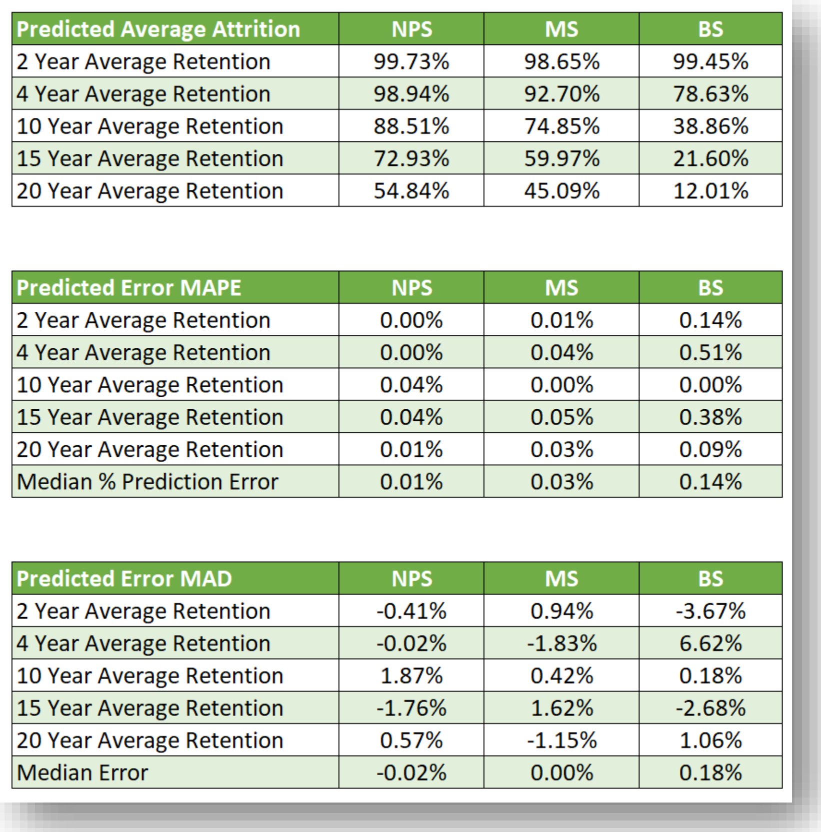
Econometric Models Ranked	R^2	p-value	Akaike AIC	Bayes BSC
Log Quadratic	0.9980	0.1539	-39.52	-40.30
Quadratic	0.9964	0.6197	5.79	5.01
Linear **	0.9958	0.0001*	6.57	5.79
Log Linear **	0.9929	0.0003*	-33.23	-34.01
Logistic **	0.9699	0.0022*	-67.68	-68.46
Linear Log	0.9216	0.0095*	21.22	20.44
Double Log	0.8756	0.0193*	-18.94	-19.72
Reciprocal	0.7232	0.0679	27.53	26.75
Log Reciprocal	0.6604	0.0946	-13.92	-14.70

Figure 22: Non-NPS MS Graduates Attrition Rate Functional Forms



Econometric Models Ranked	R^2	p-value	Akaike AIC	Bayes BSC
Quadratic	0.9950	0.0393*	12.81	12.03
Log Quadratic	0.9915	0.5715	-22.16	-22.94
Log Linear **	0.9895	0.0005*	-21.14	-21.92
Linear Log	0.9714	0.0021*	21.50	20.72
Logistic **	0.9536	0.0043*	-47.13	-47.92
Linear	0.9349	0.0072*	25.61	24.83
Double Log	0.9204	0.0098*	-10.99	-11.77
Reciprocal	0.8377	0.0294*	30.18	29.40
Log Reciprocal	0.7224	0.0682	-4.74	-5.52

Figure 23: Non-NPS BS Graduates Attrition Rate Functional Forms

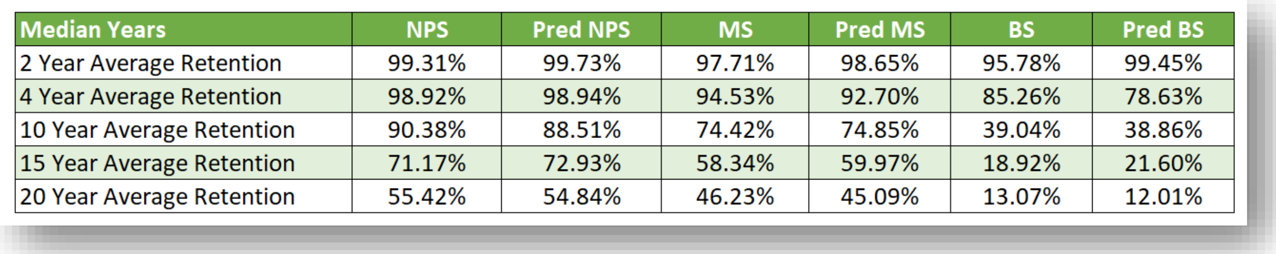


Predicted Average Attrition	NPS	MS	BS
2 Year Average Retention	99.73%	98.65%	99.45%
4 Year Average Retention	98.94%	92.70%	78.63%
10 Year Average Retention	88.51%	74.85%	38.86%
15 Year Average Retention	72.93%	59.97%	21.60%
20 Year Average Retention	54.84%	45.09%	12.01%

Predicted Error MAPE	NPS	MS	BS
2 Year Average Retention	0.00%	0.01%	0.14%
4 Year Average Retention	0.00%	0.04%	0.51%
10 Year Average Retention	0.04%	0.00%	0.00%
15 Year Average Retention	0.04%	0.05%	0.38%
20 Year Average Retention	0.01%	0.03%	0.09%
Median % Prediction Error	0.01%	0.03%	0.14%

Predicted Error MAD	NPS	MS	BS
2 Year Average Retention	-0.41%	0.94%	-3.67%
4 Year Average Retention	-0.02%	-1.83%	6.62%
10 Year Average Retention	1.87%	0.42%	0.18%
15 Year Average Retention	-1.76%	1.62%	-2.68%
20 Year Average Retention	0.57%	-1.15%	1.06%
Median Error	-0.02%	0.00%	0.18%

Figure 24: Forecast Errors of the Best Functional Form Models



Median Years	NPS	Pred NPS	MS	Pred MS	BS	Pred BS
2 Year Average Retention	99.31%	99.73%	97.71%	98.65%	95.78%	99.45%
4 Year Average Retention	98.92%	98.94%	94.53%	92.70%	85.26%	78.63%
10 Year Average Retention	90.38%	88.51%	74.42%	74.85%	39.04%	38.86%
15 Year Average Retention	71.17%	72.93%	58.34%	59.97%	18.92%	21.60%
20 Year Average Retention	55.42%	54.84%	46.23%	45.09%	13.07%	12.01%

Figure 25: Actual vs. Predicted Retention Rates (best models used)

ROI Analysis Using Cost and Benefit Lifecycle Analysis of Alternatives

Based on the two preceding subsections, we know that NPS graduates have a higher retention rate compared to non-NPS graduates (both graduate and undergraduate degree recipients), and we show that we are able to adequately predict these retention rates. Next, using these two main sources of information, we build a 20-year cost-benefit lifecycle model of a potential NPS student and future graduate, and model this officer's tenure with the Navy, compared against the prospect of not having a graduate degree or obtaining said degree at a non-military university. The cost of training a new replacement officer is the cost savings or benefits, compared to the educational cost investment required at NPS.

As mentioned, according to the *NPS Value Book,* "analysis has shown NPS to be average to below average in total costs" (Naval Postgraduate School, 2012a). Based on an internal memorandum dated January 9, 2020, NPS continuously calculates various cost-per-student measures, for Naval and reimbursable students. The 2019 NPS annual cost-per-student-full-time-equivalent (Cost/SFTE) for Naval students was approximately $42,000 per year. The NPS education cost model identifies and incorporates all costs at NPS associated with providing the academic/graduate education program. The model includes all direct costs of graduate, for-credit education as well as NPS overhead cost associated with the education. In addition, the allocated share of NPS general, administrative, and business overhead costs associated with NPS education operations and NPS MilPers costs associated with the education function may be added. However, the cost model excludes all direct costs of sponsored research activities; direct costs of executive or professional non-degree education at NPS; and the relevant allocated share of NPS general, administrative, and business overhead costs associated with NPS non-education operations such as sponsored research. In summary, the NPS cost model is broken into three points of view:

- Annual Cost-per-Student: This measure relates education costs to the effective number of full-time students onboard. Higher or lower student credit loads are not reflected. In 2019, the NPS Cost/SFTE was approximately $40,000.
- Annual Normalized-Cost/SFTE: The NPS education model provides more education and more credit hours to students than comparable civilian universities, anywhere from 50% to 100% more. Assuming an average load increase of 75%, we can normalize NPS Cost/SFTE for comparison with standard student programs at other civilian universities.

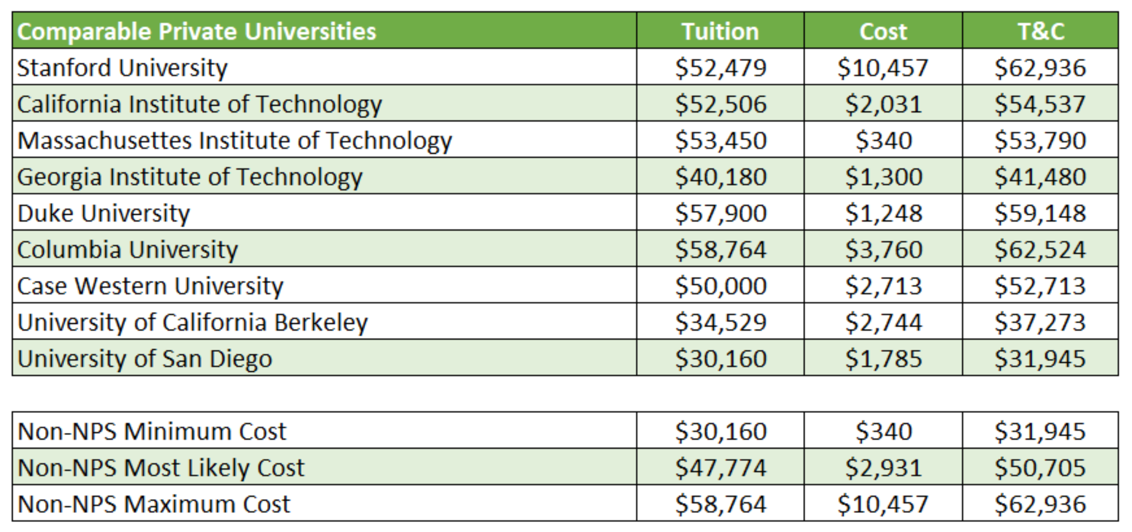
For 2019, the normalized Cost/SFTE was $34,000. NPS believes that this normalized Cost/SFTE is a more valid measure for cost comparisons.

- Annual Naval Normalized-Cost/SFTE: This is a determination of Cost-per-Student, but only for Navy Direct-funded students. In 2019, Naval Normalized-Cost/SFTE was $42,000.

For this research, the tuition costs for some comparable private universities (tuition and required cost only, excluding housing and books) were obtained, as shown in Figure 26. Next, the U.S. Treasury spot rates were obtained, and we applied a nonlinear cubic spline interpolation to generate the annualized future rates. These rates were used as the cash flow's discount rate factor to obtain the net present value of benefits and compare them with the upfront two-year educational cost (Figure 27).

A 20-year lifecycle cash flow was created using the forecasted retention rates, costs of comparable private universities, the U.S. Treasury rates, and the cost of sending a graduate student to NPS. Other expenses such as books, room and board, the officer's salary, and miscellaneous reimbursable expenses were excluded because regardless of where the Navy sends its officers, these costs would still be borne. In this research, the key consideration is the apples-to-apples relative comparison of tuition and required costs of sending a junior officer either to NPS or a non-NPS private university to obtain a graduate degree. The absolute valuation of total costs is irrelevant.

Probability distributions were set up on the cost of a private graduate degree, the NPS equivalent cost, the educational and NPS cost inflation rates, the forecasted retention rates, and the cost of training, replacement, and retention of a new officer to take the place of one who is leaving. Whenever possible, distribution fitting routines (e.g., Kolmogorov-Smirnov) were run on existing data, or theoretical metrics such as forecast standard errors were used in the simulation procedure. Simulation modeling was run using 1,000,000 trials for each input and the relevant Monte Carlo simulated net present values (NPV) and returns on investment (ROI) were computed and shown in Figures 28, 29, and 30.

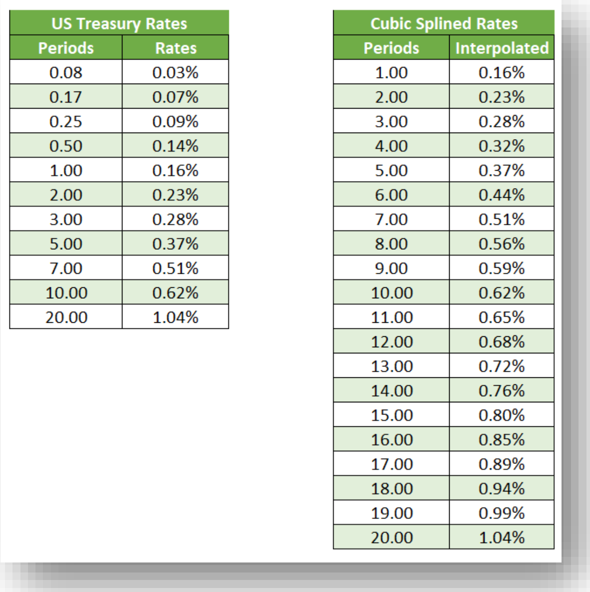


Comparable Private Universities	Tuition	Cost	T&C
Stanford University	$52,479	$10,457	$62,936
California Institute of Technology	$52,506	$2,031	$54,537
Massachusettes Institute of Technology	$53,450	$340	$53,790
Georgia Institute of Technology	$40,180	$1,300	$41,480
Duke University	$57,900	$1,248	$59,148
Columbia University	$58,764	$3,760	$62,524
Case Western University	$50,000	$2,713	$52,713
University of California Berkeley	$34,529	$2,744	$37,273
University of San Diego	$30,160	$1,785	$31,945

Non-NPS Minimum Cost	$30,160	$340	$31,945
Non-NPS Most Likely Cost	$47,774	$2,931	$50,705
Non-NPS Maximum Cost	$58,764	$10,457	$62,936

Figure 26: Cost Structure of an NPS Education Compared to Other Institutions

(*Data Sources:*
Stanford: https://registrar.stanford.edu/students-tuition
Cal Tech: http://www.gradoffice.caltech.edu/financialsupport/budget
MIT: http://gradadmissions.mit.edu/costs-funding/expenses
GA Tech: https://www.usg.edu/assets/fiscal_affairs/documents/tuition_and_fees/FY2019_Graduate_Tuition.pdf
Duke: https://gradschool.duke.edu/financial-support/cost-attend
Columbia: https://sfs.columbia.edu/tuitions-fees-listing?trf_school=382&year-period=441
Case Western: https://case.edu/studentaccounts/tuition-fees/graduateprofessional-tuition-fees
Berkeley: https://registrar.berkeley.edu/tuition-fees-residency/tuition-fees/fee-schedule
USD: https://www.sandiego.edu/finance/student-financial-services/student-accounts/cost/graduate.php)



US Treasury Rates	
Periods	Rates
0.08	0.03%
0.17	0.07%
0.25	0.09%
0.50	0.14%
1.00	0.16%
2.00	0.23%
3.00	0.28%
5.00	0.37%
7.00	0.51%
10.00	0.62%
20.00	1.04%

Cubic Splined Rates	
Periods	Interpolated
1.00	0.16%
2.00	0.23%
3.00	0.28%
4.00	0.32%
5.00	0.37%
6.00	0.44%
7.00	0.51%
8.00	0.56%
9.00	0.59%
10.00	0.62%
11.00	0.65%
12.00	0.68%
13.00	0.72%
14.00	0.76%
15.00	0.80%
16.00	0.85%
17.00	0.89%
18.00	0.94%
19.00	0.99%
20.00	1.04%

Figure 27: Treasury Discount Rates

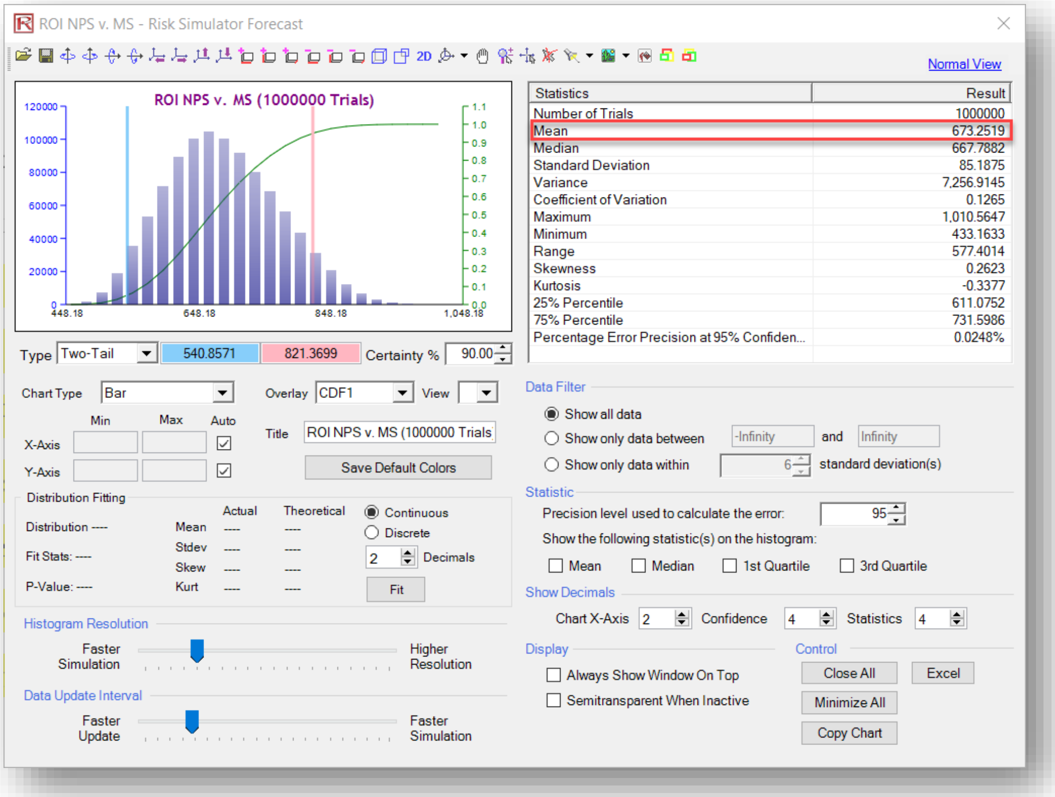
(*Source:* https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield)

A lifecycle cost model with Monte Carlo simulation was created with the following input assumptions:

- 9 comparable civilian public universities' graduate education tuition costs were obtained. The simulation assumes a triangular range of $30,160; $47,774; and $58,764.
- An annualized private education inflation rate ranging from 2.0% to 3.5% was simulated, based on the Common-fund Higher Education Price Index (HEPI).
- The relevant 1-year to 20-year U.S. Treasury rates from the U.S. Department of Treasury were used, and a nonlinear cubic spline interpolation was applied to determine the annualized forward rates. These are used as the government discount rates in the lifecycle model.
- NPS education cost used was triangulated among $34,000, $40,000, and $42,000 per year, based on the internal NPS cost model.
- NPS cost was accreted between 1.5% to 2.5% per year, based on normalized annual budgetary increases.
- The cost to train, retain, and replace a naval officer between the O-4 and O-6 levels was simulated to be between $250,000 and $500,000, depending on the billet, with a most likely cost of $350,000.
- A 20-year lifecycle model was used.
- 1,000,000 simulation trials were run in the model for the uncertain assumptions listed above and the results are shown in Figures 28, 29, and 30.

Simulation was required because every scenario and assumption above is uncertain but fluctuates within reasonable bounds. For instance, the student may decide among various alternative civilian universities (tuition costs are bounded) and may have a higher or lower attrition rate (forecast errors are bounded). Costs of education at NPS and civilian institutions can also change, but, again, within reasonable values. Finally, the inflationary rates and Treasury interest rates. Therefore, using simulation methods, we can incorporate all possible outcomes in a million scenarios of each assumption (e.g., an officer might decide on NPS vs. MIT; stay for 12 years post-graduation; happen to enroll in the two years when interest rates are the highest but the tuition rates were depressed due to low enrollment and budget cuts; and is a Navy SEAL, thereby requiring a higher replacement cost due to the specialized training requirements).

Figure 28 shows the analysis of alternatives' ROI differential when the DoD sends a junior officer to NPS for a graduate master's degree compared to sending the same officer to a private university for a similar master's degree. Due to the higher retention rates and lower costs of students who attend and graduate from NPS, we see that the expected ROI is 673%, with a 90% confidence interval of the ROI between 541% and 821%, after accounting for all the uncertainties in the input parameters and assumptions. In other words, we can safely say that, 95% of the time, given all the uncertainties and fluctuations in comparable costs and retention rates, sending an officer to NPS as compared to a private civilian graduate school will yield an additional 541% in ROI or a 6.41 return to cost ratio. Hence, for every $1 spent on an NPS education, the DoD obtains a benefit or return of $6.41 (the net benefit is $5.41 or 541%). This falls within the reasonable boundaries obtained for the ROI for naval acquisitions research programs as described in Section V.



Statistics	Result
Number of Trials	1000000
Mean	673.2519
Median	667.7882
Standard Deviation	85.1875
Variance	7,256.9145
Coefficient of Variation	0.1265
Maximum	1,010.5647
Minimum	433.1633
Range	577.4014
Skewness	0.2623
Kurtosis	-0.3377
25% Percentile	611.0752
75% Percentile	731.5986
Percentage Error Precision at 95% Confiden...	0.0248%

Type Two-Tail 540.8571 821.3699 Certainty % 90.00

Chart Type Bar
Overlay CDF1
View
Title ROI NPS v. MS (1000000 Trials

	Min	Max	Auto
X-Axis			☑
Y-Axis			☑

Distribution Fitting

		Actual	Theoretical
Distribution ----	Mean	----	----
Fit Stats: ----	Stdev	----	----
	Skew	----	----
P-Value: ----	Kurt	----	----

● Continuous
○ Discrete
2 Decimals

Data Filter
● Show all data
○ Show only data between -Infinity and Infinity
○ Show only data within 6 standard deviation(s)

Statistic
Precision level used to calculate the error: 95
Show the following statistic(s) on the histogram:
☐ Mean ☐ Median ☐ 1st Quartile ☐ 3rd Quartile

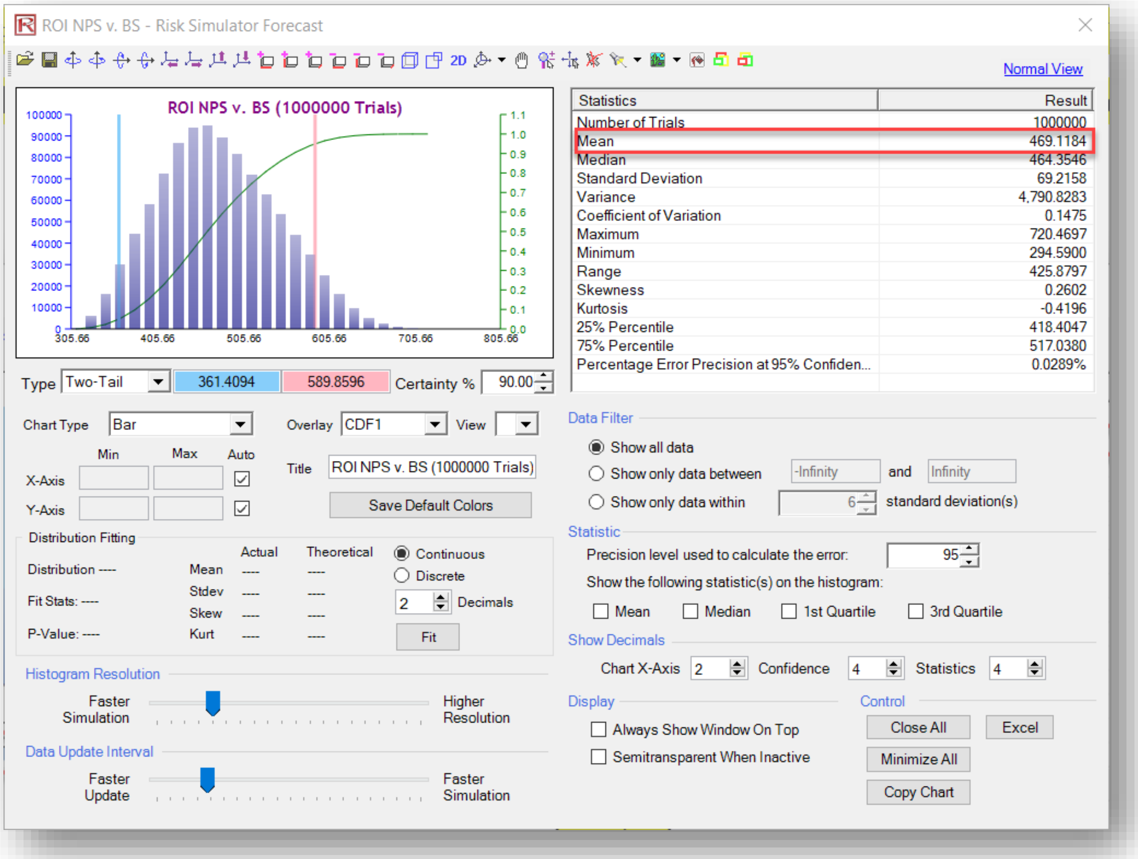
Show Decimals
Chart X-Axis 2 Confidence 4 Statistics 4

Display
☐ Always Show Window On Top
☐ Semitransparent When Inactive

Control

Figure 28: Monte Carlo Simulated ROI (NPS vs. non-NPS graduate MS)

Similarly, Figure 29 shows the analysis of alternatives' ROI differential when the DoD sends a junior officer to NPS for a graduate master's degree compared to not sending the officer at all. This situation assumes that the officer has the requisite undergraduate bachelor's degree and stays at that education level. Due to the higher retention rates of NPS graduates at the DoD, we see that the expected ROI is 469%, with a 90% confidence interval of the ROI between 361% and 590%, after accounting for all the uncertainties in the input parameters and assumptions. In other words, we can safely say that, 95% of the time, given all the uncertainties and fluctuations in NPS costs and changes in retention rates over time, sending an officer to NPS as compared to the status quo will yield an additional 361% in ROI or a 4.61 return to cost ratio. Hence, for every $1 spent on an NPS education, the DoD obtains a benefit or return of $4.61 (the net benefit is $3.61 or 361%).



Statistics	Result
Number of Trials	1000000
Mean	469.1184
Median	464.3546
Standard Deviation	69.2158
Variance	4,790.8283
Coefficient of Variation	0.1475
Maximum	720.4697
Minimum	294.5900
Range	425.8797
Skewness	0.2602
Kurtosis	-0.4196
25% Percentile	418.4047
75% Percentile	517.0380
Percentage Error Precision at 95% Confiden...	0.0289%

Type Two-Tail 361.4094 589.8596 Certainty % 90.00

Chart Type Bar
Overlay CDF1
View
Title ROI NPS v. BS (1000000 Trials)

	Min	Max	Auto
X-Axis			☑
Y-Axis			☑

Distribution Fitting

		Actual	Theoretical
Distribution ----	Mean	----	----
Fit Stats: ----	Stdev	----	----
	Skew	----	----
P-Value: ----	Kurt	----	----

● Continuous
○ Discrete
2 Decimals

Histogram Resolution
Faster Simulation — Higher Resolution

Data Update Interval
Faster Update — Faster Simulation

Data Filter
● Show all data
○ Show only data between -Infinity and Infinity
○ Show only data within 6 standard deviation(s)

Statistic
Precision level used to calculate the error: 95
Show the following statistic(s) on the histogram:
☐ Mean ☐ Median ☐ 1st Quartile ☐ 3rd Quartile

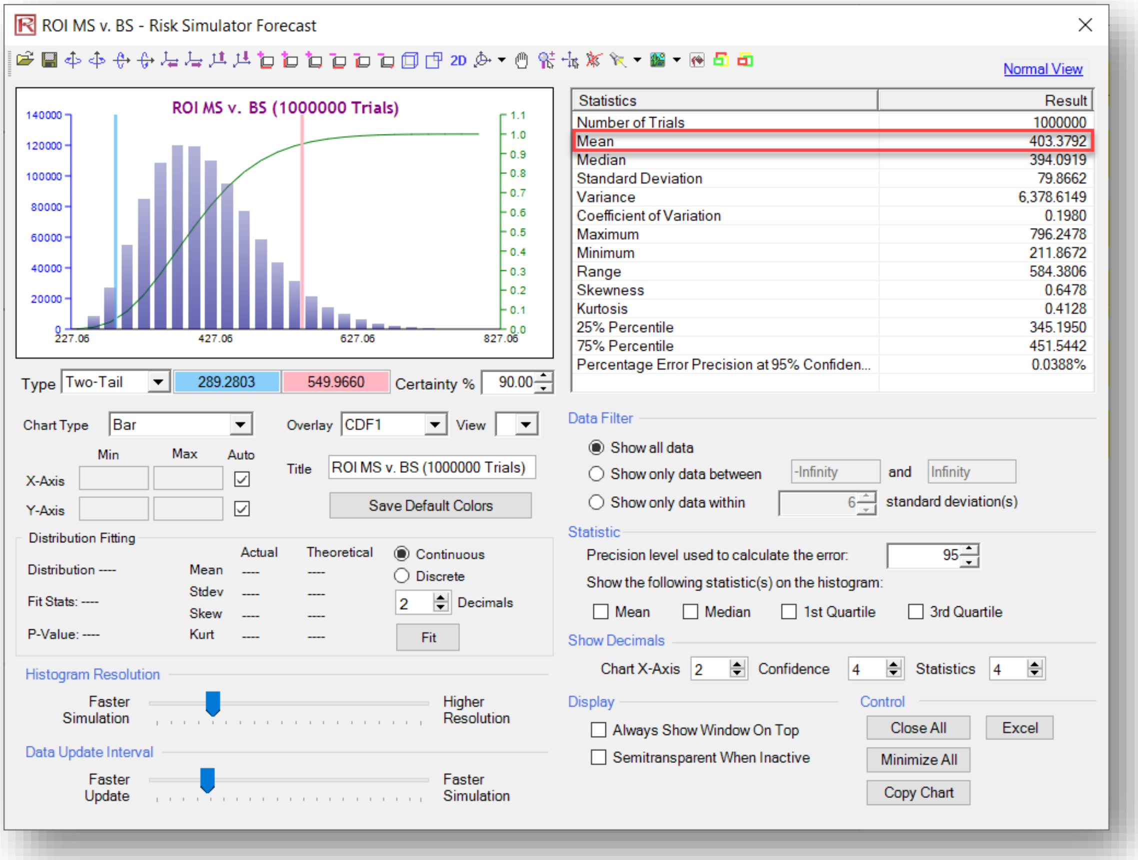
Show Decimals
Chart X-Axis 2 Confidence 4 Statistics 4

Display
☐ Always Show Window On Top
☐ Semitransparent When Inactive

Control

Figure 29: Monte Carlo Simulated ROI (NPS graduate MS vs. non-NPS undergraduate BS)

Finally, Figure 30 shows the analysis of alternatives' ROI differential when the DoD sends a junior officer to a non-NPS civilian university for a graduate master's degree compared to not sending the officer at all. This scenario again assumes that the officer has the requisite undergraduate bachelor's degree and stays at that education level. Due to the higher retention rates of graduates, we see that the expected ROI is 403%, with a 90% confidence interval of the ROI between 289% and 550%, after accounting for all the uncertainties in the input parameters and assumptions. In other words, we can safely say that, 95% of the time, given all the uncertainties and fluctuations in civilian graduate education costs and changes in retention rates over time, sending an officer to any non-NPS graduate program will yield an additional 289% in ROI or a 3.89 return to cost ratio. Hence, for every $1 spent on a non-NPS graduate education, the DoD obtains a benefit or return of $3.89 (the net benefit is $2.89 or 289%).



Statistics	Result
Number of Trials	1000000
Mean	403.3792
Median	394.0919
Standard Deviation	79.8662
Variance	6,378.6149
Coefficient of Variation	0.1980
Maximum	796.2478
Minimum	211.8672
Range	584.3806
Skewness	0.6478
Kurtosis	0.4128
25% Percentile	345.1950
75% Percentile	451.5442
Percentage Error Precision at 95% Confiden...	0.0388%

Type: Two-Tail 289.2803 549.9660 Certainty % 90.00

Chart Type: Bar
Overlay: CDF1
View:

	Min	Max	Auto
X-Axis			☑
Y-Axis			☑

Title: ROI MS v. BS (1000000 Trials)

Distribution Fitting

		Actual	Theoretical
Distribution ----	Mean	----	----
Fit Stats: ----	Stdev	----	----
	Skew	----	----
P-Value: ----	Kurt	----	----

● Continuous
○ Discrete
2 Decimals

Data Filter
● Show all data
○ Show only data between -Infinity and Infinity
○ Show only data within 6 standard deviation(s)

Statistic
Precision level used to calculate the error: 95
Show the following statistic(s) on the histogram:
☐ Mean ☐ Median ☐ 1st Quartile ☐ 3rd Quartile

Show Decimals
Chart X-Axis 2 Confidence 4 Statistics 4

Display
☐ Always Show Window On Top
☐ Semitransparent When Inactive

Control

Figure 30: Monte Carlo Simulated ROI (non-NPS graduate MS vs. non-NPS undergraduate BS)

Results Summary

In summary, we can conclude that NPS graduates show a statistically significantly higher retention rate in the U.S. Navy. Further, we can conclude that, as expected, retention rates decline over time, but the decline is fairly predictable, and the rate of decline is statistically significantly less for NPS graduates than non-NPS graduate and undergraduate degree holders. More complex econometric models with different functional forms such as logistic, loglinear, and log quadratic models were used to generate reasonable retention rates. These forecasts were then used to build lifecycle cost models and simulation models to determine the lifetime ROI for NPS students, from the point of view of a DoD investment.

We see that not only does NPS provide significant intangible value to its students and the DoD as a whole, it also provides quantifiable economic ROI. We see that from the point of view of the DoD, for every dollar invested on NPS education, the benefits return anywhere between 2.92 and 4.38 times the investment (Table 6), but, clearly, these ROI values are simply the tip of the iceberg, as the intangible value of a military graduate institution to the DoD is invaluable.

Table 6: Summary ROI for Research and Education

ROI for Military Education (e.g., NPS)	
Delta ROI: NPS vs. Civilian Master's Program (Expected Value)	673.00%
Delta ROI: NPS vs. Civilian Master's Program (90% Confidence Interval)	541%–821%
For every $1 spent on NPS, the benefit gained is $7.73 on average	
ROI: NPS Master's Program vs. Status Quo Bachelor's Degree (Expected Value)	469.00%
ROI: NPS Master's Program vs. Status Quo Bachelor's Degree (90% Confidence Interval)	361%–590%
For every $1 spent on NPS, the benefit gained is $5.69 on average	
ROI: Civilian Master's Program vs. Status Quo Bachelor's Degree (Expected Value)	403.00%
ROI: Civilian Master's Program vs. Status Quo Bachelor's Degree (90% Confidence Interval)	289%–550%
For every $1 spent on any graduate degree, the benefit gained is $5.03 on average	

Figure 31 shows the simulated ROI's probability distributions for the three scenarios. The NPS vs. civilian MS program shows the highest ROI (averaging and peaking at 673%) because the lower cost at NPS and resulting higher retention rates make it the most profitable. Second is the NPS vs. undergraduate status quo without attending any graduate programs (averaging and peaking at 469%); because the entire NPS cost is incurred, the ROI is lower than the differential cost for NPS vs. civilian MS. Finally, the lowest comparable ROI, which is still significant (averaging and peaking at 403%), is achieved when an officer attends a civilian MS program as opposed to not attending any graduate studies at all. Hence, in summary, we see that graduate education for naval officers provides significant return on the government's investment, and that NPS provides the best economic ROI, above and beyond all the qualitative and intangible values previously discussed.

These ROI values are comparable to the examples provided in the work lifecycle approach described in Section III of a civilian MBA and MS graduate of 318%, and the 304% average ROI from military research programs described in Section V. The higher ROI for NPS also results from the lower cost of education and longer retention rates of its graduates.

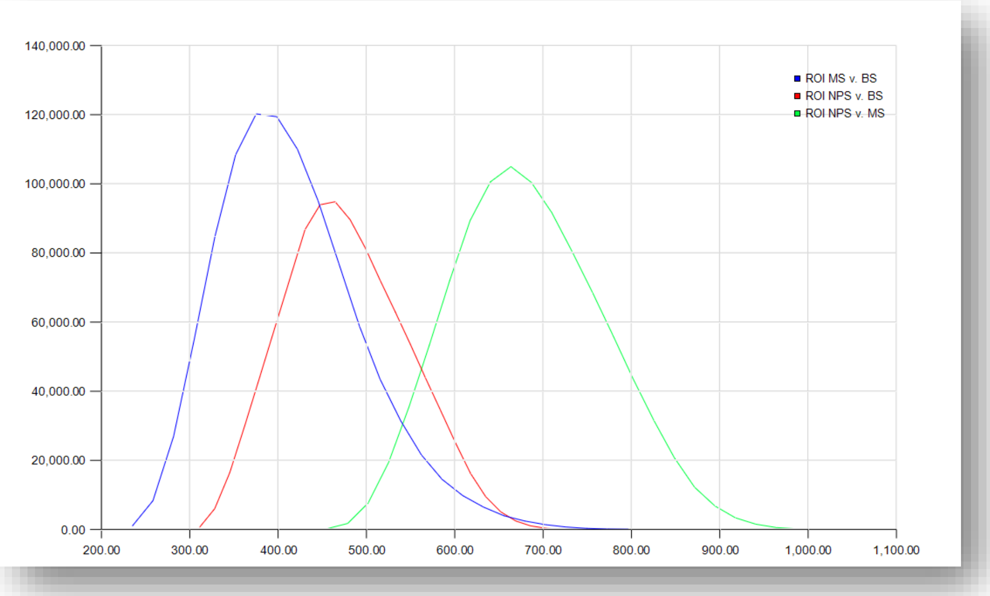


Figure 31: Overlay of Monte Carlo Simulated ROI

VII. CASE STUDY: THE VALUE AND ROI OF THE DAU

In this section, we present a brief case study of the value of the Defense Acquisition University (DAU) educational programs. The DAU is a best-in-class corporate university for the Defense Acquisition Workforce, with online courses as well as live sessions. Its mission is to provide a global learning environment to develop qualified acquisition, requirements, and contingency professionals who deliver and sustain effective and affordable warfighting capabilities (see www.dau.edu).

During FY 2017–2018, the DAU sent out surveys to tens of thousands of its course participants. These are standard end-of-course surveys taken right after the completion of a course, as well as post-course assessments that are sent as a follow-up several months later. In addition, surveys to the participants' supervisors were also submitted, several months after the conclusion of the course. The DAU uses a commercial web-based evaluation application, where some questions require a percentage response versus others requiring a 7-point Likert scale response (i.e., 1 for strongly disagree to 7 for strongly agree), to compare the survey results with other training organizations. Each year, tens to hundreds of thousands of DAU anonymous surveys are received and compared with millions of others in the database.

The surveys contain standard educational questions, including the setup of the course, the facility, quality of graded materials, quality of the faculty, and length or pace of the course. Out of the two dozen or so questions, we were able to cull the necessary data for the most relevant questions that pertain to the value of the DAU's programs. The following lists the questions selected for further analysis.

Key Questions in the Student Surveys

- VAR1. Follow-up Survey: How critical was applying the content of the training to your job success?
- VAR2. Post-event Survey: How critical is applying the content of this training to your job success?
- VAR3. Follow-up Survey: What percent of new knowledge and skills learned from this training did you directly apply to your job?
- VAR4. Post-event Survey: What percent of new knowledge and skills learned from this training do you estimate you will directly apply to your job?

- VAR5. Follow-up Survey: What percent of your total work time have you spent on tasks that require the knowledge/skills presented in the training?
- VAR6. Post-event Survey: What percent of your total work time requires the knowledge or skills presented in this training?
- VAR7. Follow-up Survey: Estimate how much of the improvement was a direct result of this training.
- VAR8. Follow-up Survey: I have been able to successfully apply the knowledge/skills learned in this class to my job.
- VAR9. Follow-up Survey: I have learned new knowledge/skills from this training.

Key Questions in the Supervisor Surveys

- VAR1: On a scale of 0% (not at all) to 100% (extremely critical), how critical is applying the content of this training to the employee's job success?
- VAR2: This training has improved the employee's job performance.
- VAR3: Given all factors, including this training, estimate how much this employee's job performance related to the course subject matter has improved since the training.
- VAR4: This training was a worthwhile investment for my organization.
- VAR5: I set expectations with this employee for this learning prior to their attending/participating in training.
- VAR6: This employee has set specific goals for using this training to do his/her job.
- VAR7: What percent of this employee's total work time do you feel he/she spends on tasks that require the knowledge/skills presented in this training?
- VAR8: What percent of new knowledge and skills learned from this training did you observe being applied by the employee to his/her job?
- VAR9: This training was a worthwhile investment in the employee's career development.
- VAR10: After training, this employee and I discussed how he/she will use the learning on his/her job.
- VAR11: I feel this employee has learned new knowledge or skills from this training.

- VAR12: Based on your response to the prior question, estimate how much of the improvement was a direct result of this training. For example, if you feel that half of the improvement was a direct result of the training, enter 50% here.
- VAR13: The employee has been able to successfully apply the knowledge/skills learned in this class to his/her job.

Of special interest is the supervisor's survey question on their view of the course's ROI. Figure 32 illustrates the results from 145 supervisors surveyed. Over 95% of the respondents would value DAU education highly, with a Likert scale of 4 or higher.

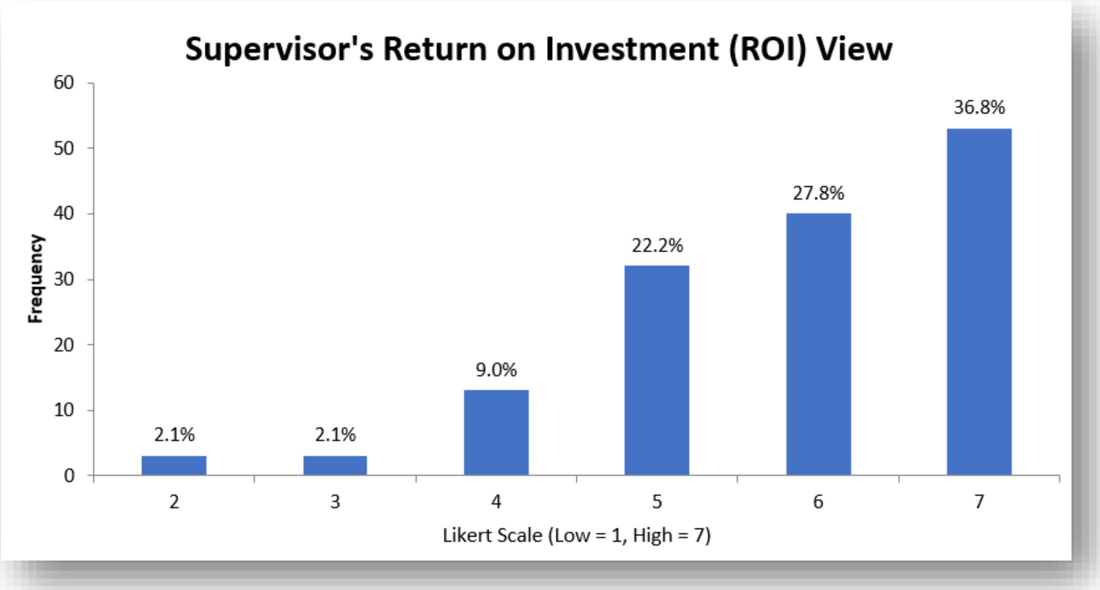


Figure 32: Perceived ROI of DAU Courses

Survey Modeling and Analysis Results

The survey results were subjected to multiple analytical models to see what critical information can be concluded from these surveys. An Inter-Class Correlation for Inter-rater Reliability Test as well as the Guttman's Lambda & Internal Consistency and Reliability Test were employed (see Appendix XIII for details) to determine if the survey responses were statistically valid, trustworthy, reliable, and replicable. In addition, econometric modeling and multivariate tests were run. Some Artificial Intelligence algorithms, such as Machine Learning, were also applied to identify any patterns that might exist in the data. The following summarizes some of the key results.

Analytical Results from Survey of Supervisors

`Inter Class Correlation for Inter-rater Reliability Test`

```
VAR1; VAR3; VAR7; VAR8; VAR12
Interclass Correlation: 0.66
Spearman-Brown Correction: 0.96
Inter-rater Reliability: 0.0000

VAR2; VAR4; VAR5; VAR6; VAR9; VAR10; VAR11; VAR13
Interclass Correlation: 0.63
Spearman-Brown Correction: 0.96
Inter-rater Reliability P-Value: 0.0000
```

`One Variable T-Test for Means`

```
VAR4, Two-Tailed P-Value: 0.0000
```

`Analysis of Variance (One Way ANOVA with Multiple Treatments)`

```
VAR1; VAR3; VAR7; VAR8; VAR12
ANOVA P-Value: 0.0000

VAR2; VAR4; VAR5; VAR6; VAR9; VAR10; VAR11; VAR13
ANOVA P-Value: 0.0000
```

`Nonparametric Kruskal-Wallis Test`

```
VAR1; VAR3; VAR7; VAR8; VAR12
Kruskal Wallis P-Value: 0.0001

VAR2; VAR4; VAR5; VAR6; VAR9; VAR10; VAR11; VAR13
Kruskal Wallis P-Value: 0.0001
```

Two Variable (T) Independent Equal Variance

```
       VAR4; VAR9          P-Value Two Tailed: 0.8021
       VAR4; VAR2          p-Value Two Tailed: 0.0058
       VAR2; VAR9          p-Value Two Tailed: 0.0022
       VAR4; VAR11         p-Value Two Tailed: 0.9592
       VAR4; VAR13         p-Value Two Tailed: 0.2287
```

Nonparametric Mann-Whitney Test

```
       VAR4; VAR9          P-Value Two Tailed: 0.9264
       VAR4; VAR2          p-Value Two Tailed: 0.0043
       VAR2; VAR9          p-Value Two Tailed: 0.0028
       VAR4; VAR11         p-Value Two Tailed: 0.7153
       VAR4; VAR13         p-Value Two Tailed: 0.1368
```

Basic Econometrics and Regression

```
Model Inputs: VAR4 vs. LN(VAR2); VAR6; VAR9; VAR13

Multiple R              0.94580     Maximum Log Likelihood            -68.84037
R-Square                0.89454     Akaike Info Criterion (AIC)         1.01849
Adjusted R-Square       0.89152     Bayes Schwarz Criterion (BSC)       1.12113
Standard Error          0.39582     Hannan-Quinn Criterion (HQC)        1.06020
Observations                145

                  Coeff     Std. Error    T-stat      P-value    Lower 5%    Upper 95%
Intercept       -0.22577     0.19891    -1.13507     0.25829    -0.61902     0.16748
LN(VAR2)         0.76059     0.19318     3.93730     0.00013     0.37867     1.14251
VAR6             0.22195     0.04354     5.09750     0.00000     0.13587     0.30803
VAR9             0.83622     0.04443    18.82177     0.00000     0.74838     0.92406
VAR13           -0.22733     0.06006    -3.78502     0.00023    -0.34608    -0.10859


ANOVA                   DF             SS             MS        F-Stat        p-Value
Regression               4         186.04          46.51     296.86391        0.00000
Residual               140          21.93           0.16
Total                  144         207.97
```

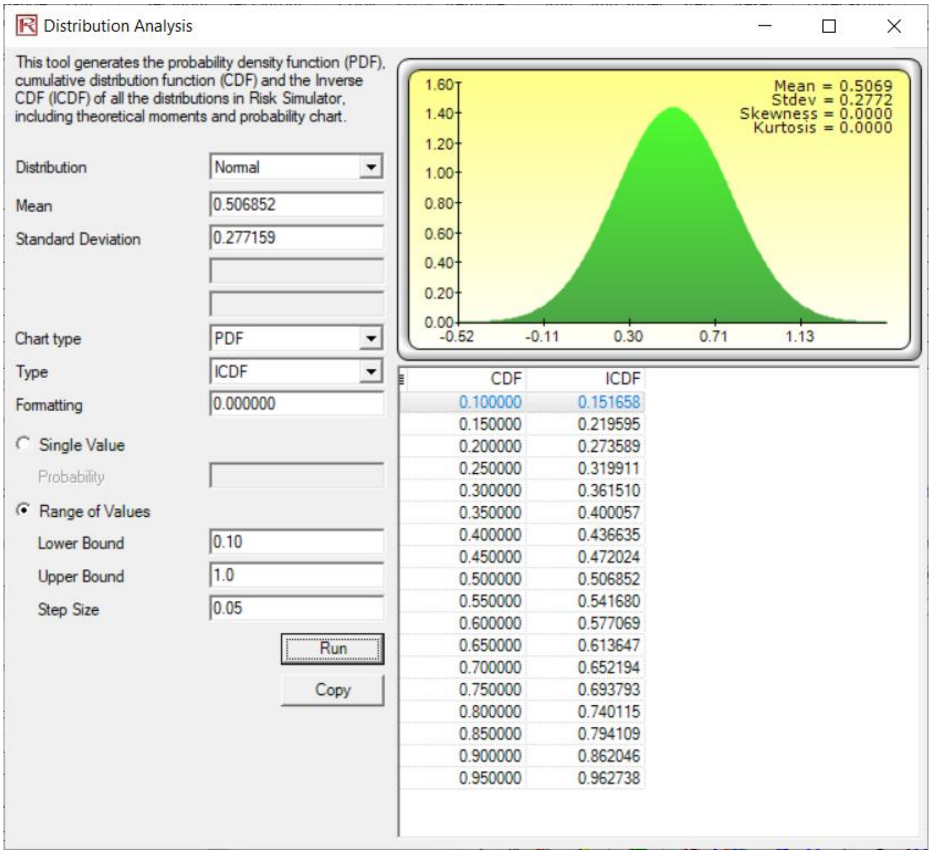
Distributional Fitting: Continuous (Anderson-Darling)

```
 Rank      MAPE %        AD    Distribution
   1      13.47%      0.1976    Normal
   2      15.37%      0.2108    Logistic
   3      16.68%      0.3170    GumbelMax
   4      27.51%      0.2899    GumbelMin

Best Fit Rank: 1
Fit Name: Normal
Anderson-Darling Statistic: 0.197647
MAPE: 0.134716
Mean: 0.506852
Sigma: 0.277159

Actual to Theoretical Four Moments:
  0.512414   0.264282   0.028672  -0.771227
  0.506852   0.277159   0.000000   0.000000
```

Conclusions from the Point of View of Supervisors

The following are the main conclusions of the DAU post-course and follow-up surveys from the point of view (POV) of the supervisors:

- There is statistical consistency and reliability among the survey responses. This means that for the 145 supervisors who sent their employees for training, their responses exhibited statistical reliability. We conclude that the responses to the survey are valid and trustworthy, rather than being completed haphazardly and without any biases. Therefore, conclusions drawn based on the survey data are statistically valid.
- We find statistical significance indicating that, on average, supervisors view that the ROI is statistically significantly greater than zero (mid-point of a Likert scale).
- Organizations value the ROI to an employee's personal career growth as being the same as the ROI to the entire organization.
- Organizations view the ROI of a training initiative to the organization as going beyond its sole impact on an employee's job performance.
- Organizations view the ROI of a training initiative to an individual employee as greater than its sole impact on an employee's job performance. This might mean that the value of training is not entirely quantifiable or immediately actionable, and that some value might be intrinsic, unmeasurable, and subjective.
- Organizations view the ROI to the organization as being more than a simple summation of actual enumerable skills or new knowledge learned. In addition, organizations perceive ROI as being more than applications of specific knowledge or skill set on the job.
- Organizations see value if the training helped improved an employee's performance and enabled the employee to successfully apply the knowledge and skills, but only if it is also worthwhile to the employee's own career development based on specific goals and expectations set prior to the training course. Each of these criteria by itself does not necessarily contribute to the perceived ROI but only when they are combined holistically.
- Using distributional fitting, we see that the probability distribution of the estimated improvement percentage as a direct result of a training course (VAR12) shows that, on average, there is a 50.7% increase in productivity (Figure 33), with three quarters of the supervisors surveyed saying that productivity improvements were at least 32%.



This tool generates the probability density function (PDF), cumulative distribution function (CDF) and the Inverse CDF (ICDF) of all the distributions in Risk Simulator, including theoretical moments and probability chart.

Distribution: Normal
Mean: 0.506852
Standard Deviation: 0.277159
Chart type: PDF
Type: ICDF
Formatting: 0.000000
○ Single Value
Probability
● Range of Values
Lower Bound: 0.10
Upper Bound: 1.0
Step Size: 0.05

CDF	ICDF
0.100000	0.151658
0.150000	0.219595
0.200000	0.273589
0.250000	0.319911
0.300000	0.361510
0.350000	0.400057
0.400000	0.436635
0.450000	0.472024
0.500000	0.506852
0.550000	0.541680
0.600000	0.577069
0.650000	0.613647
0.700000	0.652194
0.750000	0.693793
0.800000	0.740115
0.850000	0.794109
0.900000	0.862046
0.950000	0.962738

Figure 33: Probability Distribution of Supervisors' POV

Analytical Results from Survey of Students

Inter Class Correlation for Inter-rater Reliability Test

```
VAR1; VAR2; VAR3; VAR4; VAR5; VAR6; VAR7
Interclass Correlation: 0.33
Spearman-Brown Correction: 0.93
Inter-rater Reliability: 0.0000

VAR1; VAR3; VAR5; VAR7
Interclass Correlation: 0.74
Spearman-Brown Correction: 0.93
Inter-rater Reliability P-Value: 0.0000

VAR2; VAR4; VAR6
Interclass Correlation: 0.84
Spearman-Brown Correction: 0.96
Inter-rater Reliability P-Value: 0.0000
```

```
       VAR8; VAR9
       Interclass Correlation: 0.02
       Spearman-Brown Correction: 0.04
       Inter-rater Reliability P-Value: 0.0032
```

Analysis of Variance (One Way ANOVA with Multiple Treatments)

```
       VAR1; VAR2; VAR3; VAR4; VAR5; VAR6; VAR7
       ANOVA P-Value: 0.0000
```

Nonparametric Kruskal-Wallis Test

```
       VAR1; VAR2; VAR3; VAR4; VAR5; VAR6; VAR7
       Kruskal Wallis P-Value: 0.0000
```

Two Variable (T) Independent Equal Variance

```
       VAR1; VAR2          P-Value Two Tailed: 0.0000
       VAR3; VAR4          p-Value Two Tailed: 0.0000
       VAR5; VAR6          p-Value Two Tailed: 0.0000
```

Nonparametric Mann-Whitney Test

```
       VAR1; VAR2          P-Value Two Tailed: 0.0000
       VAR3; VAR4          p-Value Two Tailed: 0.0000
       VAR5; VAR6          p-Value Two Tailed: 0.0000
```

Basic Econometrics and Stepwise Regression

```
ARRANGEMENT: Y<->X3;X7;X1;X5

Regression Results

OVERALL FIT
Multiple R              0.75271      Maximum Log Likelihood            3753.34608
R-Square                0.56658      Akaike Info Criterion (AIC)         -0.46393
Adjusted R-Square       0.56647      Bayes Schwarz Criterion (BSC)       -0.45964
Standard Error          0.19180      Hannan-Quinn Criterion (HQC)        -0.46251
Observations              16142


                   Coeff  Std. Error      T-stat     P-value    Lower 5%   Upper 95%
Intercept       -0.01274     0.00614    -2.07418     0.03808    -0.02479    -0.00070
VAR3             0.37981     0.01040    36.50831     0.00000     0.35942     0.40020
VAR8             0.02617     0.00132    19.87973     0.00000     0.02359     0.02875
VAR1             0.25276     0.01001    25.24347     0.00000     0.23314     0.27239
VAR5             0.05341     0.00968     5.51641     0.00000     0.03443     0.07239


ANOVA
                      DF          SS          MS           F     p-Value
Regression             4      776.01      194.00  5273.63619     0.00000
Residual           16137      593.63        0.04
Total              16141     1369.64

Hypothesis Test
  Critical F-statistic (99% confidence with DFR1 and DFR2): 3.320336
```

```
  Critical F-statistic (95% confidence with DFR1 and DFR2): 2.372483
  Critical F-statistic (90% confidence with DFR1 and DFR2): 1.945208
```

Random Forest Supervised Data Mining

```
Bagging with 100 iterations and base learner with Cross-validation

Correlation coefficient                  0.8659
Mean absolute error                      0.0923
Root mean squared error                  0.1470
Relative absolute error                 37.356%
Root relative squared error             50.091%
Total Number of Instances                16,142
```

k-Means Clustering

```
Number of iterations: 19
Within cluster sum of squared errors: 8084.545176982922

Initial starting points (random):

Cluster 0: 1,0.8,1,0.7,0.5,0.6,0.5,7,7
Cluster 1: 0.1,0.1,0.1,0.1,0.1,0.1,0.1,5,6

Missing values globally replaced with mean/mode

Final cluster centroids:
                               Cluster#
Attribute     Full Data        0          1
              (16142.0)    (8044.0)   (8098.0)
============================================
VAR1             0.4754     0.7143     0.2382
VAR2             0.5071     0.5019     0.5123
VAR3             0.4385     0.6783     0.2003
VAR4             0.4941     0.4882     0.5000
VAR5             0.4060     0.6141     0.1994
VAR6             0.4555     0.4495     0.4616
VAR7             0.4398     0.6393     0.2416
VAR8             5.5050     6.2471     4.7678
VAR9             5.6808     5.6875     5.6742
```

Artificial Intelligence Multi-Layered Perceptron

```
Classifier model (full training set)

Linear Node 0
    Inputs          Weights
    Threshold       0.06925846705171
    Node 1          -0.9353491867299
    Node 2          1.00459405724956
    Node 3          1.58048358855907
    Node 4          -0.8778430933414
```

Distributional Fitting: Continuous (Anderson-Darling)

```
Rank       MAPE %           AD    Distribution
   1       45.80%       0.2826    GumbelMax
   2       46.98%       0.4680    Fréchet
```

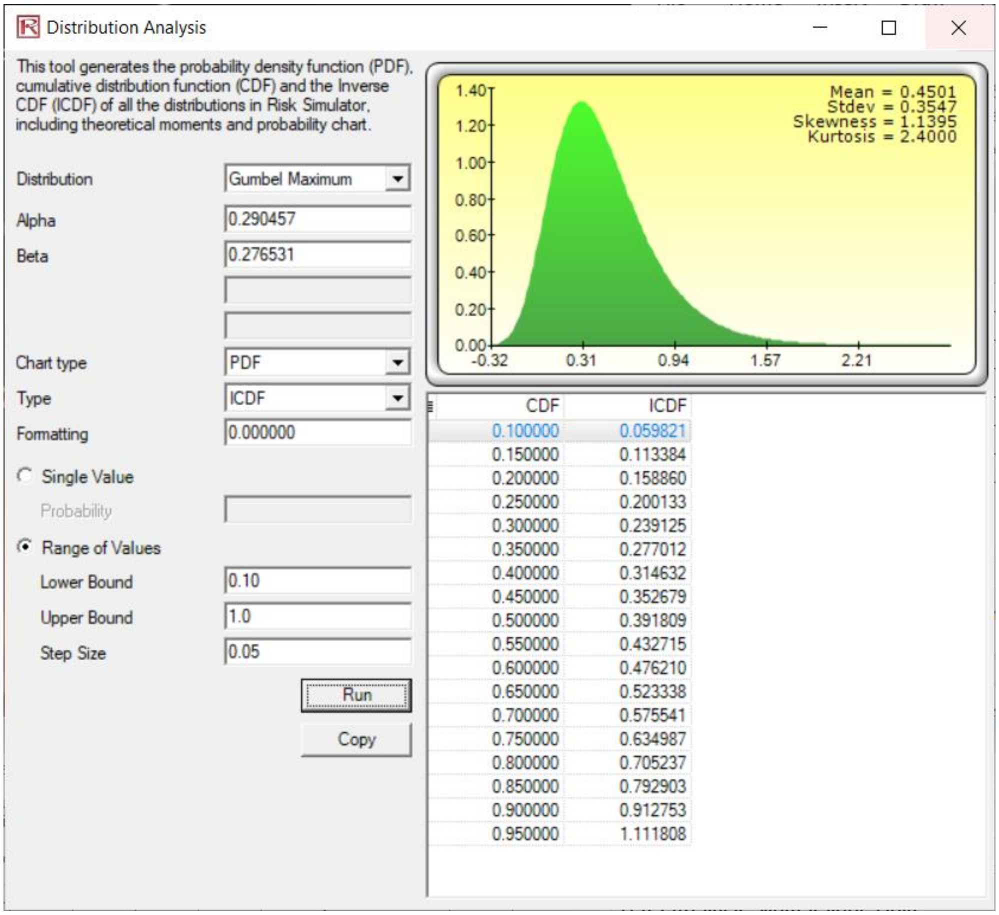
```
   3      53.94%      0.2703    Normal
   4      57.65%      0.2782    Logistic
   5      88.72%      0.3492    GumbelMin
   6     289.64%      0.7048    TDist
   7     447.11%      1.0000    Standard Normal
   8     477.33%      1.0758    Weibull3
   9     551.54%      0.4355    Exponential2
  10    2710.10%         N/A    Uniform

Best Fit Rank: 1
Fit Name: GumbelMax
Alpha: 0.290457
Anderson-Darling Statistic: 0.282634
Beta: 0.276531
MAPE: 0.458042

Actual to Theoretical Four Moments:
  0.439753   0.291298   0.234879  -0.931816
  0.450074   0.354664   1.139547   2.400000
```

Conclusions from the Point of View of Students

The following are the main conclusions of the DAU post-course and follow-up surveys from the point of view (POV) of the students:

- For the 16,157 students who responded to the surveys, the responses as a whole exhibited statistical reliability as well as statistical consistency, indicating that there were no biases in the data. We can conclude that the responses to the survey are valid and trustworthy, rather than being completed haphazardly. Therefore, conclusions drawn based on the survey data are statistically valid.
- The student’s view at the end of the course in terms of the usefulness of the course material presented is materially and significantly different after spending time on the job.
- The student’s view at the end of the course in terms of the amount of new knowledge learned that might be applicable to their job is materially and significantly different after spending time on the job.
- The student’s view at the end of the course in terms of the amount of work time requiring the use of the new knowledge learned is materially and significantly different after spending time on the job.
- There is a statistically significant improvement the student’s work abilities as a direct result of the training received.

- There is a statistically significant increase in the ability to apply the knowledge and skills learned in class.
- There is a statistically significant amount of new knowledge learned in class.
- At a future follow-up session, a former student's estimate of how much work improvement was a direct result of the training course depended on actual experience during the follow-up session and are not completely known immediately after the course ended.
- Figure 34 shows that about three-quarters of the students surveyed believed that their productivity increased at least 20% after taking the course. When compared against the supervisor's POV, we see that the students' POV (Gumbel distribution) is more conservative than the supervisors' POV (normal distribution), but with similar shape and scale (Figure 35).



This tool generates the probability density function (PDF), cumulative distribution function (CDF) and the Inverse CDF (ICDF) of all the distributions in Risk Simulator, including theoretical moments and probability chart.

Distribution: Gumbel Maximum
Alpha: 0.290457
Beta: 0.276531
Chart type: PDF
Type: ICDF
Formatting: 0.000000
○ Single Value
Probability:
● Range of Values
Lower Bound: 0.10
Upper Bound: 1.0
Step Size: 0.05

Mean = 0.4501
Stdev = 0.3547
Skewness = 1.1395
Kurtosis = 2.4000

CDF	ICDF
0.100000	0.059821
0.150000	0.113384
0.200000	0.158860
0.250000	0.200133
0.300000	0.239125
0.350000	0.277012
0.400000	0.314632
0.450000	0.352679
0.500000	0.391809
0.550000	0.432715
0.600000	0.476210
0.650000	0.523338
0.700000	0.575541
0.750000	0.634987
0.800000	0.705237
0.850000	0.792903
0.900000	0.912753
0.950000	1.111808

Figure 34: Probability Distribution of Students' POV

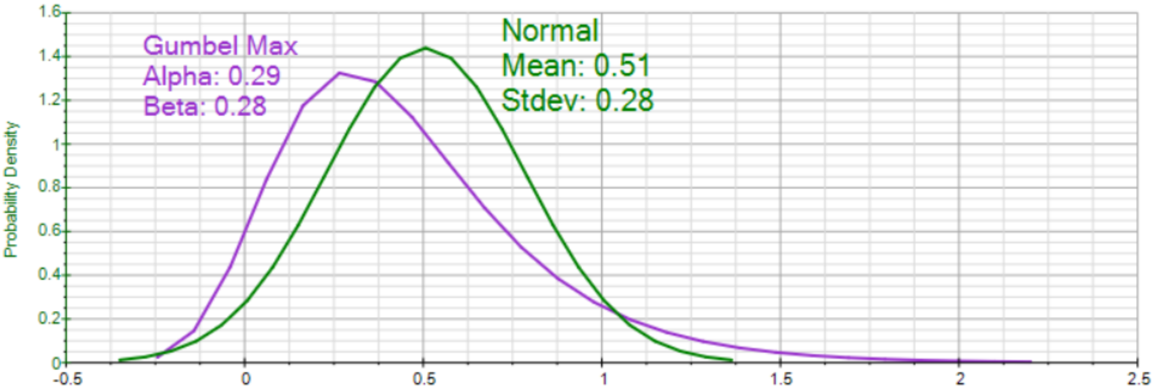


Figure 35: Comparing the Student vs. Supervisor POV

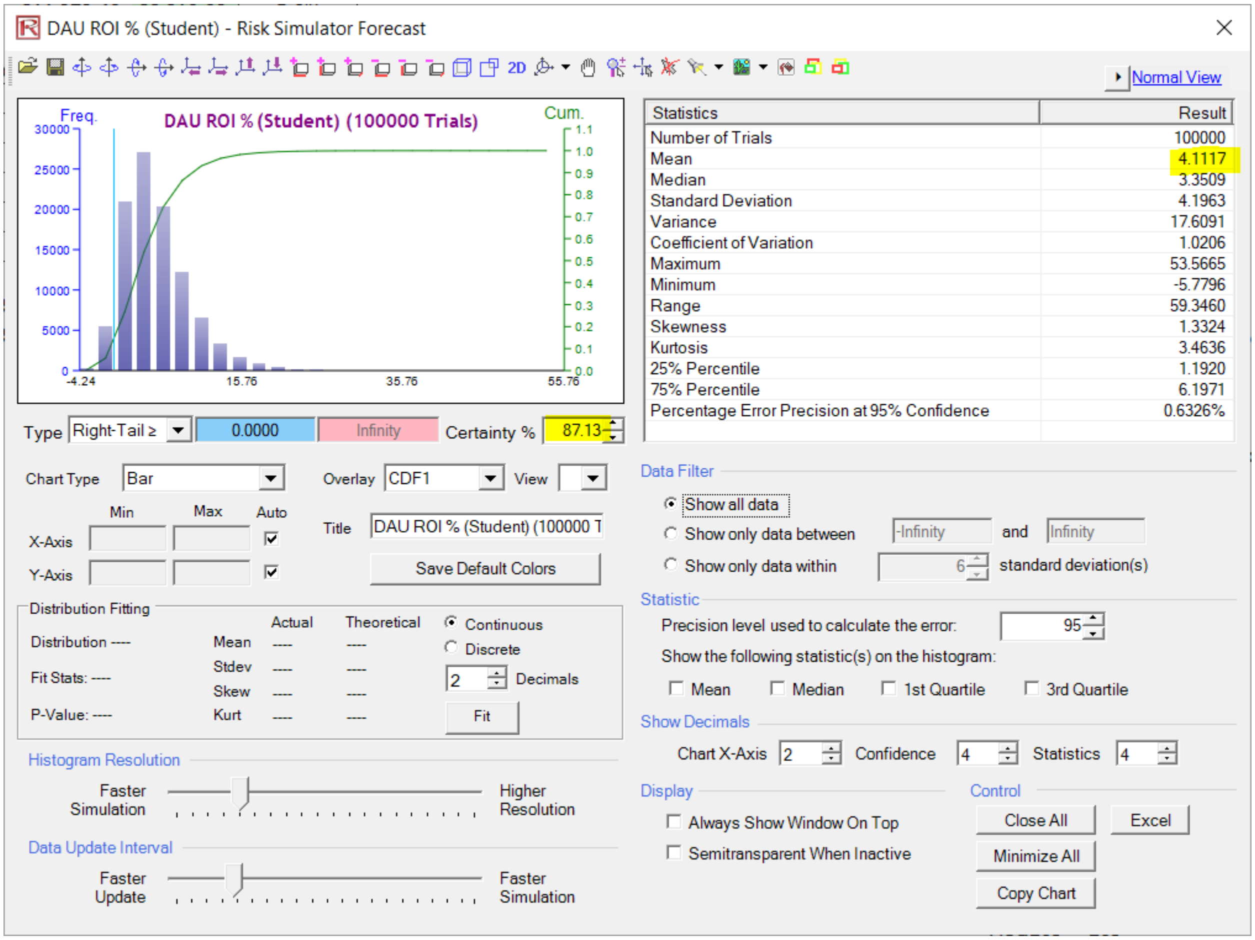
Return on Investment Analysis

Finally, an analysis of the return on investment (ROI) is performed on the DAU courses. Several assumptions are made to enable the ROI analysis, namely:

- We used the DAU's own annual report to determine that there are over 152,557 students taking online courses and 44,326 graduates from resident courses in FY 2019 (source: https://www.dau.edu/about/Documents/AnnualReport.pdf).
- The FY 2020 Congressional Budget request was for $163 million, which covers all operating costs of the DAU, including any requisite travel expenses for its students, faculty salaries, operations and maintenance of its facilities, and other expenses.
- The average cost per student, averaged across online and resident programs, is between $900 and $4500. The lower end applies to mostly online courses versus resident courses at the upper end of the range, as well as varying depending on the course type and course level.
- Based on the survey of over 16,157 students, there were 171 different courses, and the allocation of these course levels (100-, 200-, 300-, and 400-level courses) are unequally distributed among O-1 to O-6 officers (we excluded special seminars for Flag officers), with the predominant number of students at the O-3 to O-5 levels, spread across multiple 100- and 200-level courses.

- Using the O-1 to O-6 pay scales, and assuming that the faculty members are between GS-12 and GS-15 levels, a Monte Carlo risk simulation was run to determine the cost of education for an average course (source: www.federalpay.org).
- Similarly, probability distributional and curve-fitting routines were run on the perceived enhanced efficiency and effectiveness at doing one's job, as determined from the six-month follow-up surveys. Using these distributions, Monte Carlo risk simulations were run to determine the potential ROI.

The conclusion is that the average ROI from the POV of the students and supervisors/organizations are between **411%** and **477%**, and the probability that, on average, any given course taken at the DAU has at least **87%** and **93%** probabilities that the ROI is positive, from the POV of the student and the supervisor/organization, respectively (Figures 36 and 37).



DAU ROI % (Student) - Risk Simulator Forecast

Statistics	Result
Number of Trials	100000
Mean	4.1117
Median	3.3509
Standard Deviation	4.1963
Variance	17.6091
Coefficient of Variation	1.0206
Maximum	53.5665
Minimum	-5.7796
Range	59.3460
Skewness	1.3324
Kurtosis	3.4636
25% Percentile	1.1920
75% Percentile	6.1971
Percentage Error Precision at 95% Confidence	0.6326%

Type Right-Tail ≥ 0.0000 Infinity Certainty % 87.13

Chart Type Bar
Overlay CDF1
View
X-Axis Min Max Auto ☑
Y-Axis Min Max Auto ☑
Title DAU ROI % (Student) (100000 T

Distribution Fitting
Distribution ----
Fit Stats: ----
P-Value: ----

	Actual	Theoretical
Mean	----	----
Stdev	----	----
Skew	----	----
Kurt	----	----

● Continuous
○ Discrete
2 Decimals

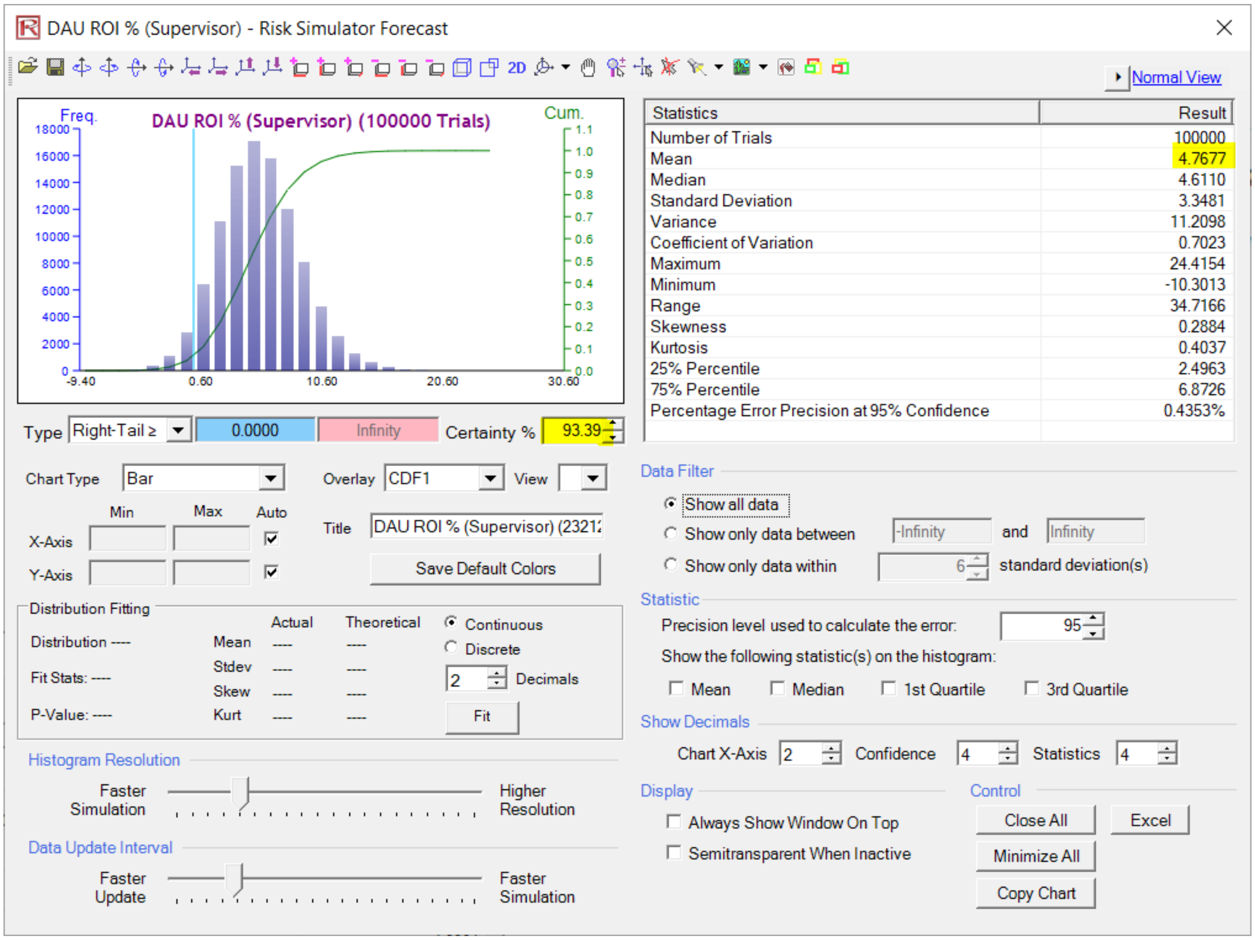
Data Filter
● Show all data
○ Show only data between -Infinity and Infinity
○ Show only data within 6 standard deviation(s)

Statistic
Precision level used to calculate the error: 95
Show the following statistic(s) on the histogram:
☐ Mean ☐ Median ☐ 1st Quartile ☐ 3rd Quartile

Show Decimals
Chart X-Axis 2 Confidence 4 Statistics 4

Display
☐ Always Show Window On Top
☐ Semitransparent When Inactive

Figure 36: Simulated Students' ROI



Statistics	Result
Number of Trials	100000
Mean	4.7677
Median	4.6110
Standard Deviation	3.3481
Variance	11.2098
Coefficient of Variation	0.7023
Maximum	24.4154
Minimum	-10.3013
Range	34.7166
Skewness	0.2884
Kurtosis	0.4037
25% Percentile	2.4963
75% Percentile	6.8726
Percentage Error Precision at 95% Confidence	0.4353%

Type Right-Tail ≥ 0.0000 Infinity Certainty % 93.39

Chart Type Bar Overlay CDF1 View

	Min	Max	Auto
X-Axis			☑
Y-Axis			☑

Title DAU ROI % (Supervisor) (2321

Distribution Fitting

		Actual	Theoretical
Distribution ----	Mean	----	----
Fit Stats: ----	Stdev	----	----
	Skew	----	----
P-Value: ----	Kurt	----	----

● Continuous
○ Discrete
2 Decimals

Histogram Resolution

Data Update Interval

Data Filter

● Show all data
○ Show only data between -Infinity and Infinity
○ Show only data within 6 standard deviation(s)

Statistic

Precision level used to calculate the error: 95

Show the following statistic(s) on the histogram:

☐ Mean ☐ Median ☐ 1st Quartile ☐ 3rd Quartile

Show Decimals

Chart X-Axis 2 Confidence 4 Statistics 4

Display

☐ Always Show Window On Top
☐ Semitransparent When Inactive

Control

Figure 37: Simulated Supervisors’ ROI

VIII. CONCLUSION

As the basis for reorienting education, the U.S. Department of the Navy (2018), through the E4S report, recommends the following strategic vision: "The Naval Education Enterprise must produce leaders of character, integrity, and intelligence steeped not only in the art of war, the profession of arms, and the history and traditions of the Naval service, but also in a broader understanding of the technical and strategic complexities of the Cognitive Age, vital to assuring success in war, peace, and grey zone conflict; officer and enlisted leaders of every rank who think critically, communicate clearly, and are imbued with a bias for decisive and ethical action."

As such, the motivation for the main research question was whether military education and research has any value to the DON and DoD in general, and if so, how would one compute its ROI. We consider the fact that the drive for lifelong education in naval officers are personal and also an institutional responsibility. Education is vital for the strategic viability and long-term lethality of our warfighting forces and country.

In the E4S report, a survey of past Naval students at NPS, NWC, and USNA indicated that approximately 96% agreed that formal education was extremely useful or very useful in their naval careers. In fact, the study found that military personnel have more positive perceptions of their institutions than civilian personnel. In addition, the faculty at USNA perceives their institution as "better at preparing naval officers to be more effective leaders, excel in their fields of study, apply their education to real world situations, establish and manage effective teams, and understand critical strategies significantly better" than if they attended civilian institutions (Department of the Navy, 2018).

We can certainly conclude that the intangible value of military education is significant, in developing leadership and critical thinking skills for junior as well as senior officers. The military-oriented curriculum taught by faculty members with former military experience or knowledge allows the flow of institutional knowledge down to the students. Although these intangible and qualitative side of military education is significant, this current research focuses on the more quantitative measure of ROI.

Using NPS as a case study, we can further conclude that NPS graduates show a statistically significantly higher retention rates in the U.S. Navy. Further, we can conclude that as expected, retention rates decline over time, but the decline is fairly predictable, and the rate of decline is statistically significantly less for NPS graduates than non-NPS graduate degree holders and undergraduate degree holders. More complex econometric models with different functional forms such as logistic, loglinear, log quadratic models were used to generate reasonable retention rates. These forecasts were then used to build lifecycle cost models and simulation models to determine the lifetime ROI for NPS students, from the point of view of a DoD investment. Finally, Machine Learning algorithms in Artificial Intelligence were also applied for pattern recognition purposes.

Table 7 recaps the critical results from the research. From the ROI computed, we can unequivocally conclude that a graduate education at NPS provides statistically significantly higher retention rates, which eventually translates to a high positive ROI to the DoD.

In Section V, we saw that the ROI for military-based research has significant qualitative intangible worth as well as quantitative economic ROI. In summary, we can quantify that the ARP's ROI based on an annual investment of $1.7M will range from the absolute worst case of 121% to an average of **240%–600%** for each specific program. The KVA method pegs the ROI at 253%. Therefore, using standard industry best practices, we conclude the average conservative ROI for the entire ARP program to be approximately **304%**.

In Section VI, the analysis was extended to look at the ROI of NPS. We see that from the point of view of the DoD, for every dollar invested in NPS education, the benefits return anywhere between 5.7 and 7.7 times the investment, which represents expected ROIs between **469%** and **673%** (Table 7). These ROI values are miniscule in comparison to the holistic, intangible, and qualitative value of a military graduate university to the DoD.

In Section VII, using the Defense Acquisition University data, we determine that the ROI of military education in the acquisitions world is between **411%** and **477%**, and the probability that, on average, any given course taken at the DAU has at least **87%** and **93%** probabilities that the ROI is positive.

In conclusion, the global average for DOD education, on average, provides the government an ROI of approximately **485%**.

Table 7: Summary ROI for Research and Education

ROI for Military Research and Development (e.g., ARP)	
Minimal Worst-Case ROI	121.00%
Most Likely ROI	304.00%
Range of ROI Depending on Program	240%–600%

ROI for Military Education (e.g., NPS)	
Delta ROI: NPS vs. Civilian Master’s Program (Expected Value)	673.00%
Delta ROI: NPS vs. Civilian Master’s Program (90% Confidence Interval)	541%–821%
For every $1 spent on NPS, the benefit gained is $7.73 on average	
ROI: NPS Master’s Program vs. Status Quo Bachelor’s Degree (Expected Value)	469.00%
ROI: NPS Master’s Program vs. Status Quo Bachelor’s Degree (90% Confidence Interval)	361%–590%
For every $1 spent on NPS, the benefit gained is $5.69 on average	
ROI: Civilian Master’s Program vs. Status Quo Bachelor’s Degree (Expected Value)	403.00%
ROI: Civilian Master’s Program vs. Status Quo Bachelor’s Degree (90% Confidence Interval)	289%–550%
For every $1 spent on any graduate degree, the benefit gained is $5.03 on average	
ROI for Short or Specialized Military Courses (e.g., DAU)	
ROI on DAU Courses on Average	411%–477%
For every $1 spent on DAU, the benefit gained is $5.77 on average	
Global Average ROI (ARP, NPS, DAU): 485%	

Limitations and Recommendations for Future Research

This research examined and created various theoretical constructs and empirical methods to generate ROI for military education and research. The current research both proposes these methodologies and used available data to simulate cash flow lifecycle models. The recommended next steps of the research would be to obtain long-term data from current and previous students via survey instruments, interviews, work performance data, and other requisite information that flows out of this data collection process. The data with higher fidelity can then be reprocessed through the methodologies described.

In order to facilitate the execution of the proposed methodologies in this research project, the recommendation is to apply the following research instruments, which will require institutional review board (IRB) authorization. Clearly, the efforts listed next can evolve over time based on the results obtained throughout the research project. Nonetheless, suffice to say, the effort involved going forward should be a multiyear research program. Therefore, given the time and budgetary constraints inherent in this current research, the following represents our current research's limitations and opportunities for the future.

- **Better Cost Data.** Higher-level precision cost data would be helpful in clearly identifying the actual ROI. The costs at NPS need to be stratified and segmented into the correct subcategories to make them comparable to private civilian universities. In addition, the cost of attrition and benefits of retention starting from the career of a junior officer through senior officer ranks needs to be better quantified. Other costs would be the opportunity costs of empty billets at the senior officer and flag or general officer levels.

- **Surveys.** Sliding scale surveys of past graduate students at NPS and NWC. These surveys, coupled with performance reviews, can be used to run the Analytical Framework Approach recommended in this research.

- **Focus Groups**. Qualitative and intangible value to society, service, and the nation can be culled from such focus group discussions. Anecdotal evidence can be obtained and extrapolated to incorporate other pockets of evidence of intangible value of military education and research.

- **Follow-up Questionnaires**. These can be numerical in nature and are based on the responses from the initial surveys and focus group results. The questionnaire can also be used to follow up on previously determined anecdotal events.

- **On-the-job Observations.** These observations can be performed if a certain learned skill or applied research is put into action. The number of times a certain skill is triggered (frequency of use), and the impact (impact amplitude) to the overall process can be noted. The Frequency and Quantity of Used method can be applied to capture the frequency and amplitude of knowledge use. The information captured can then be fed into the Knowledge Value Added methodology and Monte Carlo Simulated to determine the potential impact on cost reduction and efficiency increase in having the research or knowledge applied.

- **Performance Reviews.** Multiple year performance review of graduates before and after their education program, as well as a random selection of performance reviews of other officers with similar billets and rank, which can be used as a control group. Using these hard data, meta-analysis can be performed, and both the Empirical Impact Approach and Analytical Framework Approach can be applied to the numerical hard data. This is similar to a controlled test when comparing before-after effects and with-without effects of education and research.

- **Tracking Over Time.** The graduates' career over time should be tracked, including promotions and billet changes over time.

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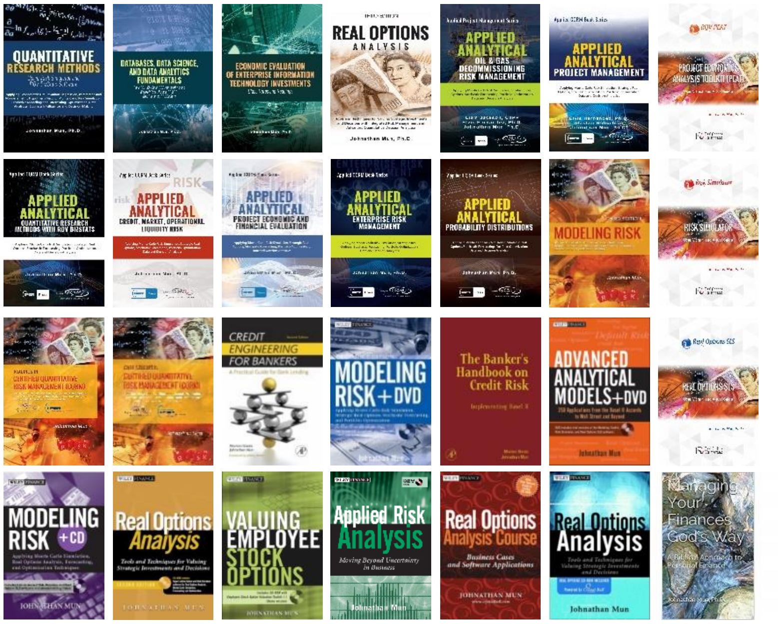
IX. BIOGRAPHY

JOHNATHAN MUN, Ph.D.

Research Professor
Naval Postgraduate School
Information Sciences Department
Glasgow West, Monterey, CA 93943
Email: jcmun@nps.edu



Dr. Johnathan C. Mun is a research professor at the U.S. Naval Postgraduate School (Monterey, California) and teaches master's and doctoral courses as well as executive seminars in quantitative risk analysis, decision sciences, real options, stochastic simulation, portfolio optimization, and other related concepts. He has also researched and consulted on many Department of Defense and Department of Navy projects and is considered a leading world expert on risk analysis and real options analysis.

By the numbers, Dr. Mun has over 11 registered patents and 10 patents pending; authored and published 28 books translated into 5 languages; written book chapters in over 16 books; and published over 60 articles in academic journals, symposia, and proceedings. His books include diverse areas such as *Quantitative Research Methods,* (ROV Press 2019), *Databases, Data Science and Data Analytics Fundamentals,* (IIPER Press 2019), *Modeling Risk: Applying Monte Carlo Simulation, Real Options, Optimization, and Forecasting,* First, Second, and Third Editions (Wiley 2006, Wiley 2010, and Thomson-Shore 2015); *Real Options Analysis: Tools and Techniques,* First, Second, and Third Editions (Wiley 2003, Wiley 2005, and Thomson-Shore 2016); *Advanced Analytical Models* (Wiley 2008 and ROV Press 2016); *The Banker's Handbook on Credit and Market Risk* (Elsevier Science 2008); and others. He is the creator of the following software: *Real Options Super Lattice Solver*, *Risk Simulator*, *Project Economics Analysis Tool* (PEAT), *Modeling Toolkit*, *Risk Explorer*, and *ESO Valuation*. His books and software are being used at top universities around the world (including the Bern Institute in Germany, Chung-Ang University in South Korea, Georgetown University, ITESM in Mexico, Massachusetts Institute of Technology, Naval Postgraduate School, New York University, Stockholm University in Sweden, University of the Andes in Chile, University of Chile, University of Pennsylvania Wharton School, University of Hull in the United Kingdom, and Edinburgh University in Scotland).

Dr. Mun has taught at universities all over the world, from the U.S. Naval Postgraduate School (Monterey, California) and University of Applied Sciences (Switzerland and Germany) as full professor, to Golden Gate University (California) and St. Mary's College (California), and has chaired many graduate research thesis committees. He also teaches risk analysis, real options analysis, and risk analysis for managers public courses where participants can obtain the Certified in Quantitative Risk Management (CQRM) designation on completion of the week-long program. He also holds the position of the EU President of the American Academy of Financial Management and sits on the Global Board of Standards at the AAFM. He is the founder and currently the CEO of Real Options Valuation, Inc., and was formerly the Vice President of Analytics at Crystal Ball/Decisioneering, Inc., where he headed the development of options and financial analytics software products, analytical consulting, training, and technical support. Prior to that, he was a Consulting Manager and Financial Economist in the Valuation Services and Global Financial Services practice of KPMG Consulting and a Manager with the Economic Consulting Services practice at KPMG LLP. He has extensive experience in econometric modeling, financial analysis, real options, economic analysis, and statistics. He has consulted on a variety of real options, risk analysis, financial forecasting, project management, and financial valuation issues for over 100 multinational firms (former clients include 3M, Airbus, Boeing, BP, Chevron Texaco, Financial Accounting Standards Board, Fujitsu, GE, Microsoft, Motorola, Pfizer, Timken, U.S. Department of Defense, U.S. Navy, Veritas, and many others). His experience prior to joining KPMG included being Department Head of financial planning and analysis at Viking Inc. of FedEx, performing financial forecasting, economic analysis, and research.

Dr. Mun received his Ph.D. in Finance and Economics from Lehigh University, where his research and academic interests were in the areas of Investment Finance, Econometric Modeling, Financial Options, Corporate Finance, and Microeconomic Theory. He also has an M.B.A. in business administration, an M.S. in management science, and a B.S. in Biology and Physics. He is Certified in Financial Risk Management (FRM), Certified in Financial Consulting (CFC), and Certified in Quantitative Risk Management (CQRM). He is a member of the American Mensa, Phi Beta Kappa Honor Society, and Golden Key Honor Society as well as several other professional organizations, including the Eastern and Southern Finance Associations, American Economic Association, and Global Association of Risk Professionals. Finally, he has written many academic articles published in the *Advances in Quantitative Finance and Accounting, Global Finance Journal, International Financial Review,* the *Journal of Financial Analysis, Journal of Applied Financial Economics, Journal of International Financial Markets, Institutions and Money, Financial Engineering News, Journal of the Society of Petroleum Engineers, Naval Engineers Journal*, *Journal of Reliability Engineering and System Safety*, *Computers & Operations Research*, *Defense Acquisition Research Journal*, *International Journal of Finance and Economics*, *Journal for Money and Banking*, *Journal of Economic Strategies*, *Systems Engineering Research*, *Review of Business Research*, *International Review of Financial Analysis*, and *NPS Acquisitions Research Program*.

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XI. APPENDIX I: NPS Hall of Fame

NPS ALUMNI HALL OF FAME

Vice Admiral Jan E. Tighe, USN

The Honorable Jack R. Borsting

The Honorable Everett Alvarez

General Keith B. Alexander, USA (Ret.)

Mr. Walt Havenstein

Admiral Eric T. Olson, USN (Ret.)

Admiral Stanley Arthur, USN (Ret.)

Dr. Jack London

Vice Admiral Thomas J. Hughes, USN (Ret)

Admiral T. Joseph Lopez, USN (Ret)

Vice Admiral Pat Tracey, USN (Ret)

General Apichart Penkitti

Permanent Secretary for Defense, Thailand

Admiral Mike Mullen, CJCS, USN

General Michael Hagee, USMC (Ret.)

The Honorable Dan Albert

Admiral Wayne E. Meyer, USN (Ret.)

Admiral James D. Watkins, USN (Ret.)

General John A. Gordon, USAF (Ret.)

Admiral Henry H. Mauz, Jr., USN (Ret.)

Vice Admiral Arthur K. Cebrowski, USN (Ret.)

Professor Lui Pao Chuen

The Honorable James G. Roche

The Honorable Thomas E. White

DISTINGUISHED ALUMNI AWARDS

GEN Keith Alexander, USA

ADM Stanley Arthur, USN (Ret)

Col Walter H. Augustin, USMC (Ret)

CAPT Jeffrey Bacon, USN (Ret)

VADM Roger F. Bacon, USN (Ret)

VADM Phillip Balisle, USN

Arthur H. Barber, III

VADM John T. "Terry" Blake, USN (Ret)

RADM Stanley Bozin, USN

RADM Michael A. Brown, USN

VADM Nancy E. Brown, USN

VADM William A. Brown, USN

CAPT Daniel W. Bursch, USN (Ret)

CAPT Todd Calhoun, USMC (Ret)

VADM Arthur Cebrowski, USN

CDR Sandra K. Chachula, USN (Ret)

RDML Philip J. Coady Jr., USN (Ret)

RADM Dan W. Davenport, USN

RDML Patrick W. Dunne, USN

CMDR Gordon Eubanks, USN (Ret)

VADM Mark E. Ferguson, III, USN

CAPT Stephen Frick, USN

RADM William J. Galinis, USN

RADM James B Greene Jr., USN (Ret)

VADM Lee F. Gunn, USN (Ret)

RADM Charles S. Hamilton II, USN

ADM Cecil D. Haney, USN

LTG David K. Heebner, USA (Ret)

RADM Elizabeth A. Hight, USN

RADM Jon A. Hill, USN

Col David Hilmers, USMC (Ret)

CAPT Sam Houston, USN (Ret)

VADM P. Gardner Howe III, USN

VADM Thomas J. Hughes, USN (Ret)

CAPT Wayne P. Hughes, Jr., USN (Ret)

VADM Harvey E. Johnson, Jr., USCG (Ret)

RADM John M. Kelly, USN

LtGen Richard S. Kramlich, USMC

RADM William Landay III, USN

LCDR Marvin Langston, USN (Ret)

Chief Cathy Lanier (Ret), Washington, D.C. Police

CAPT Donald M. Layton, USN (Ret)

LtGen Chan Lee, ROKAF

RADM Michael A. LeFever, USN

RADM Richard Lewis, USN

VADM Keith W. Lippert, USN

CAPT Michael Lopez-Alegria, USN

Hon. Michael D. Lumpkin

RADM Archer M. Macy, Jr., USN

VADM Dr. Desi Mamahit, Indonesian Navy

RADM Michael Mathis, USN (Ret)

VADM Justin McCarthy SC, USN

RDML Timothy J. McGee, USN

ADM William McRaven, USN

RADM Wayne Meyer, USN

VADM Michael Mullen, USN

ADM Robert J. Natter, USN (Ret)

LtCol Carlos Noriega, USMC (Ret)

ADM Eric T. Olson, USN

Dr. Michael A. Parker, USAF

CAPT Alan Poindexter, USN

VADM John Scott Redd, USN, (Ret)

CAPT Kenneth Reightler, Jr., USN (Ret)

The Honorable James Roche, Captain, USN (Ret)

VADM Marcelo Barreto Rodrigues, Brazil Navy

RADM Conrad J. Rorie, USN (Ret)

VADM Ronald A. Route, USN (Ret)

VADM Almir Garnier Santos
Federal Republic of Brazil Navy

CDR Carter "Buzz" Savage, USN (Ret)

CAPT Dylan Schmorrow, USN

CAPT Winston Scott, USN (Ret)

RADM Kenneth Slaght, USN

VADM Stanley Szemborski, USN

RDML (Sel) Jan Tighe, USN

VADM Patricia A. Tracey, USN (Ret)

BRIG GEN Alice Trevino, USAF

LTG Thomas R. Turner, USA

VADM Scott R. Van Buskirk, USN (Ret)

MG Michael A. Vane, USA

VADM Michael C. Vitale, USN (Ret)

MG Kirk F. Vollmecke, USA

GEN William S. Wallace, USA

LTG Eric P. Wendt, USA

The Honorable Thomas White,
Secretary of the Army

COL Jeff Williams, USA (Ret)

RDML Jesse A. Wilson, Jr., USN

RADM Edward Winters, III, USN

The Honorable Robert O. Work
Deputy Secretary of Defense

CAPT Janice Wynn, USN

CAPT John A. Zangardi, USN (Ret)

XII. APPENDIX II: Analytical Results

One Way ANOVA Models for Single Factor Multiple Treatments

```
One Way ANOVA with Randomized Multiple Treatments

Model Inputs:
VAR1; VAR2; VAR3
NPS, MS, BS

One Way ANOVA with Randomized Multiple Treatments

                      DF    Sums of Squares    Mean Square      F Stat    p-Value
Between Groups         2              56.35          28.17     10.2180     0.0006
Within Groups         24              66.17           2.76
Total                 26             122.52           4.71

F Critical @ 0.10                  2.538332
F Critical @ 0.05                  3.402826
F Critical @ 0.01                  5.613592


--------------------------------------------------------------------------------------

Model Inputs:
VAR5; VAR6; VAR7
NPS, MS, BS

One Way ANOVA with Randomized Multiple Treatments

                      DF    Sums of Squares    Mean Square      F Stat    p-Value
Between Groups         2             876.35         438.18     30.6794     0.0000
Within Groups         24             342.78          14.28
Total                 26            1219.13          46.89

F Critical @ 0.10                  2.538332
F Critical @ 0.05                  3.402826
F Critical @ 0.01                  5.613592


--------------------------------------------------------------------------------------

Model Inputs:
VAR9; VAR10; VAR11
NPS, MS, BS

One Way ANOVA with Randomized Multiple Treatments

                      DF    Sums of Squares    Mean Square      F Stat    p-Value
Between Groups         2           16158.95        8079.47    291.3330     0.0000
Within Groups         24             665.59          27.73
Total                 26           16824.53         647.10

F Critical @ 0.10                  2.538332
F Critical @ 0.05                  3.402826
F Critical @ 0.01                  5.613592
```

```
Model Inputs:
VAR13; VAR14; VAR15
NPS, MS, BS

One Way ANOVA with Randomized Multiple Treatments

                      DF    Sums of Squares    Mean Square      F Stat    p-Value
Between Groups         2           13205.23        6602.61    228.3266     0.0000
Within Groups         24             694.02          28.92
Total                 26           13899.25         534.59

F Critical @ 0.10                  2.538332
F Critical @ 0.05                  3.402826
F Critical @ 0.01                  5.613592


--------------------------------------------------------------------------------------

Model Inputs:
VAR17; VAR18; VAR19
NPS, MS, BS

One Way ANOVA with Randomized Multiple Treatments

                      DF    Sums of Squares    Mean Square      F Stat    p-Value
Between Groups         2            5955.77        2977.88     17.8394     0.0001
Within Groups         15            2503.91         166.93
Total                 17            8459.68         497.63

F Critical @ 0.10                  2.695173
F Critical @ 0.05                  3.682320
F Critical @ 0.01                  6.358874


--------------------------------------------------------------------------------------
```

Nonparametric Pairwise Kruskal Wallis Test

```
Model Inputs:
VAR1; VAR2; VAR3
NPS, MS, BS


             Kruskal-Wallis Test
H Statistic: 15.844797
p-Value: 0.000363
H Critical at 1%: 9.210340
H Critical at 5%: 5.991465
H Critical at 10%: 4.605170
The population medians are statistically not equal at 1%, 5%, or 10% significance.

-------------------------------------------------------------------------------------

Model Inputs:
VAR5; VAR6; VAR7
NPS, MS, BS


             Kruskal-Wallis Test
H Statistic: 21.188713
p-Value: 0.000025
H Critical at 1%: 9.210340
H Critical at 5%: 5.991465
H Critical at 10%: 4.605170
The population medians are statistically not equal at 1%, 5%, or 10% significance.
Model Inputs:
VAR9; VAR10; VAR11
NPS, MS, BS


             Kruskal-Wallis Test
H Statistic: 23.142857
p-Value: 0.000009
H Critical at 1%: 9.210340
H Critical at 5%: 5.991465
H Critical at 10%: 4.605170
The population medians are statistically not equal at 1%, 5%, or 10% significance.

-------------------------------------------------------------------------------------

Model Inputs:
VAR13; VAR14; VAR15
NPS, MS, BS


             Kruskal-Wallis Test
H Statistic: 22.317460
p-Value: 0.000014
H Critical at 1%: 9.210340
H Critical at 5%: 5.991465
H Critical at 10%: 4.605170
The population medians are statistically not equal at 1%, 5%, or 10% significance.

-------------------------------------------------------------------------------------
```

```
Model Inputs:
VAR17; VAR18; VAR19
NPS, MS, BS


             Kruskal-Wallis Test
H Statistic: 11.941520
p-Value: 0.002552
H Critical at 1%: 9.210340
H Critical at 5%: 5.991465
H Critical at 10%: 4.605170
The population medians are statistically not equal at 1%, 5%, or 10% significance.

---------------------------------------------------------------------------------------
```

Parametric Two Variable Independent Variables T Test with Unequal Variances

```
Model Inputs:
VAR1; VAR2
NPS, MS


             Two Variable (T) Independent Unequal Variance
Column 1 Observations: 9
Column 1 Sample Mean: 99.311111
Column 1 Sample Standard Deviation: 0.862329
Column 2 Observations: 9
Column 2 Sample Mean: 97.711111
Column 2 Sample Standard Deviation: 2.213281
Sample Mean Difference: 1.600000
t-Statistic: 2.020767
Hypothesized Mean: 0.000000

p-Value Left Tailed: 0.964555
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly less than the hypothesized mean difference.

p-Value Right Tailed: 0.035445
significant at 10% and 5%
rejected
significantly greater than the hypothesized mean difference.

p-Value Two Tailed: 0.070890
significant at 10%
rejected
significantly different than the hypothesized mean difference.

---------------------------------------------------------------------------------------
```

```
Model Inputs:
VAR1; VAR3
NPS, BS


             Two Variable (T) Independent Unequal Variance
Column 1 Observations: 9
Column 1 Sample Mean: 99.311111
Column 1 Sample Standard Deviation: 0.862329
Column 2 Observations: 9
Column 2 Sample Mean: 95.777778
Column 2 Sample Standard Deviation: 1.621556
Sample Mean Difference: 3.533333
t-Statistic: 5.771572
Hypothesized Mean: 0.000000

p-Value Left Tailed: 0.999956
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly less than the hypothesized mean difference.

p-Value Right Tailed: 0.000044
significant at 1%, 5% and 10%
rejected
significantly greater than the hypothesized mean difference.

p-Value Two Tailed: 0.000089
significant at 1%, 5% and 10%
rejected
significantly different than the hypothesized mean difference.
Model Inputs:
VAR2; VAR3
MS, BS


             Two Variable (T) Independent Unequal Variance
Column 1 Observations: 9
Column 1 Sample Mean: 97.711111
Column 1 Sample Standard Deviation: 2.213281
Column 2 Observations: 9
Column 2 Sample Mean: 95.777778
Column 2 Sample Standard Deviation: 1.621556
Sample Mean Difference: 1.933333
t-Statistic: 2.113910
Hypothesized Mean: 0.000000

p-Value Left Tailed: 0.974158
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly less than the hypothesized mean difference.

p-Value Right Tailed: 0.025842
significant at 10% and 5%
rejected
significantly greater than the hypothesized mean difference.

p-Value Two Tailed: 0.051683
significant at 10%
rejected
significantly different than the hypothesized mean difference.

--------------------------------------------------------------------------------------

Model Inputs:
```

```
VAR5; VAR6
NPS, MS


             Two Variable (T) Independent Unequal Variance
Column 1 Observations: 9
Column 1 Sample Mean: 98.922222
Column 1 Sample Standard Deviation: 0.884276
Column 2 Observations: 9
Column 2 Sample Mean: 94.533333
Column 2 Sample Standard Deviation: 3.511766
Sample Mean Difference: 4.388889
t-Statistic: 3.635808
Hypothesized Mean: 0.000000

p-Value Left Tailed: 0.997282
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly less than the hypothesized mean difference.

p-Value Right Tailed: 0.002718
significant at 1%, 5% and 10%
rejected
significantly greater than the hypothesized mean difference.

p-Value Two Tailed: 0.005436
significant at 1%, 5% and 10%
rejected
significantly different than the hypothesized mean difference.
Model Inputs:
VAR5; VAR7
NPS, BS


             Two Variable (T) Independent Unequal Variance
Column 1 Observations: 9
Column 1 Sample Mean: 98.922222
Column 1 Sample Standard Deviation: 0.884276
Column 2 Observations: 9
Column 2 Sample Mean: 85.255556
Column 2 Sample Standard Deviation: 5.452777
Sample Mean Difference: 13.666667
t-Statistic: 7.422140
Hypothesized Mean: 0.000000

p-Value Left Tailed: 0.999963
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly less than the hypothesized mean difference.

p-Value Right Tailed: 0.000037
significant at 1%, 5% and 10%
rejected
significantly greater than the hypothesized mean difference.

p-Value Two Tailed: 0.000075
significant at 1%, 5% and 10%
rejected
significantly different than the hypothesized mean difference.

--------------------------------------------------------------------------------------
```

```
Model Inputs:
VAR6; VAR7
MS, BS


             Two Variable (T) Independent Unequal Variance
Column 1 Observations: 9
Column 1 Sample Mean: 94.533333
Column 1 Sample Standard Deviation: 3.511766
Column 2 Observations: 9
Column 2 Sample Mean: 85.255556
Column 2 Sample Standard Deviation: 5.452777
Sample Mean Difference: 9.277778
t-Statistic: 4.291443
Hypothesized Mean: 0.000000

p-Value Left Tailed: 0.999627
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly less than the hypothesized mean difference.

p-Value Right Tailed: 0.000373
significant at 1%, 5% and 10%
rejected
significantly greater than the hypothesized mean difference.

p-Value Two Tailed: 0.000746
significant at 1%, 5% and 10%
rejected
significantly different than the hypothesized mean difference.
Model Inputs:
VAR9; VAR10
NPS, MS


             Two Variable (T) Independent Unequal Variance
Column 1 Observations: 9
Column 1 Sample Mean: 90.377778
Column 1 Sample Standard Deviation: 3.430298
Column 2 Observations: 9
Column 2 Sample Mean: 74.422222
Column 2 Sample Standard Deviation: 6.442588
Sample Mean Difference: 15.955556
t-Statistic: 6.558069
Hypothesized Mean: 0.000000

p-Value Left Tailed: 0.999987
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly less than the hypothesized mean difference.

p-Value Right Tailed: 0.000013
significant at 1%, 5% and 10%
rejected
significantly greater than the hypothesized mean difference.

p-Value Two Tailed: 0.000027
significant at 1%, 5% and 10%
rejected
significantly different than the hypothesized mean difference.

--------------------------------------------------------------------------------------
```

```
Model Inputs:
VAR9; VAR11
NPS, BS


             Two Variable (T) Independent Unequal Variance
Column 1 Observations: 9
Column 1 Sample Mean: 90.377778
Column 1 Sample Standard Deviation: 3.430298
Column 2 Observations: 9
Column 2 Sample Mean: 32.377778
Column 2 Sample Standard Deviation: 5.470324
Sample Mean Difference: 58.000000
t-Statistic: 26.947971
Hypothesized Mean: 0.000000

p-Value Left Tailed: 1.000000
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly less than the hypothesized mean difference.

p-Value Right Tailed: 0.000000
significant at 1%, 5% and 10%
rejected
significantly greater than the hypothesized mean difference.

p-Value Two Tailed: 0.000000
significant at 1%, 5% and 10%
rejected
significantly different than the hypothesized mean difference.
Model Inputs:
VAR10; VAR11
MS, BS


             Two Variable (T) Independent Unequal Variance
Column 1 Observations: 9
Column 1 Sample Mean: 74.422222
Column 1 Sample Standard Deviation: 6.442588
Column 2 Observations: 9
Column 2 Sample Mean: 32.377778
Column 2 Sample Standard Deviation: 5.470324
Sample Mean Difference: 42.044444
t-Statistic: 14.924003
Hypothesized Mean: 0.000000

p-Value Left Tailed: 1.000000
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly less than the hypothesized mean difference.

p-Value Right Tailed: 0.000000
significant at 1%, 5% and 10%
rejected
significantly greater than the hypothesized mean difference.

p-Value Two Tailed: 0.000000
significant at 1%, 5% and 10%
rejected
significantly different than the hypothesized mean difference.

-------------------------------------------------------------------------------------
```

```
Model Inputs:
VAR13; VAR14
NPS, MS


             Two Variable (T) Independent Unequal Variance
Column 1 Observations: 9
Column 1 Sample Mean: 71.166667
Column 1 Sample Standard Deviation: 3.576660
Column 2 Observations: 9
Column 2 Sample Mean: 57.444444
Column 2 Sample Standard Deviation: 7.366667
Sample Mean Difference: 13.722222
t-Statistic: 5.027048
Hypothesized Mean: 0.000000

p-Value Left Tailed: 0.999852
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly less than the hypothesized mean difference.

p-Value Right Tailed: 0.000148
significant at 1%, 5% and 10%
rejected
significantly greater than the hypothesized mean difference.

p-Value Two Tailed: 0.000296
significant at 1%, 5% and 10%
rejected
significantly different than the hypothesized mean difference.
Model Inputs:
VAR13; VAR15
NPS, BS


             Two Variable (T) Independent Unequal Variance
Column 1 Observations: 9
Column 1 Sample Mean: 71.166667
Column 1 Sample Standard Deviation: 3.576660
Column 2 Observations: 9
Column 2 Sample Mean: 18.922222
Column 2 Sample Standard Deviation: 4.437561
Sample Mean Difference: 52.244444
t-Statistic: 27.499427
Hypothesized Mean: 0.000000

p-Value Left Tailed: 1.000000
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly less than the hypothesized mean difference.

p-Value Right Tailed: 0.000000
significant at 1%, 5% and 10%
rejected
significantly greater than the hypothesized mean difference.

p-Value Two Tailed: 0.000000
significant at 1%, 5% and 10%
rejected
significantly different than the hypothesized mean difference.

--------------------------------------------------------------------------------------
```

```
Model Inputs:
VAR14; VAR15
MS, BS


             Two Variable (T) Independent Unequal Variance
Column 1 Observations: 9
Column 1 Sample Mean: 57.444444
Column 1 Sample Standard Deviation: 7.366667
Column 2 Observations: 9
Column 2 Sample Mean: 18.922222
Column 2 Sample Standard Deviation: 4.437561
Sample Mean Difference: 38.522222
t-Statistic: 13.438010
Hypothesized Mean: 0.000000

p-Value Left Tailed: 1.000000
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly less than the hypothesized mean difference.

p-Value Right Tailed: 0.000000
significant at 1%, 5% and 10%
rejected
significantly greater than the hypothesized mean difference.

p-Value Two Tailed: 0.000000
significant at 1%, 5% and 10%
rejected
significantly different than the hypothesized mean difference.
Model Inputs:
VAR17; VAR18
NPS, MS


             Two Variable (T) Independent Unequal Variance
Column 1 Observations: 6
Column 1 Sample Mean: 55.416667
Column 1 Sample Standard Deviation: 17.054667
Column 2 Observations: 6
Column 2 Sample Mean: 46.233333
Column 2 Sample Standard Deviation: 13.619202
Sample Mean Difference: 9.183333
t-Statistic: 1.030660
Hypothesized Mean: 0.000000

p-Value Left Tailed: 0.836508
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly less than the hypothesized mean difference.

p-Value Right Tailed: 0.163492
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly greater than the hypothesized mean difference.

p-Value Two Tailed: 0.326983
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly different than the hypothesized mean difference.

-------------------------------------------------------------------------------------
```

```
Model Inputs:
VAR17; VAR19
NPS, BS


             Two Variable (T) Independent Unequal Variance
Column 1 Observations: 6
Column 1 Sample Mean: 55.416667
Column 1 Sample Standard Deviation: 17.054667
Column 2 Observations: 6
Column 2 Sample Mean: 13.066667
Column 2 Sample Standard Deviation: 4.943548
Sample Mean Difference: 42.350000
t-Statistic: 5.842071
Hypothesized Mean: 0.000000

p-Value Left Tailed: 0.999446
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly less than the hypothesized mean difference.

p-Value Right Tailed: 0.000554
significant at 1%, 5% and 10%
rejected
significantly greater than the hypothesized mean difference.

p-Value Two Tailed: 0.001109
significant at 1%, 5% and 10%
rejected
significantly different than the hypothesized mean difference.
Model Inputs:
VAR18; VAR19
MS, BS


             Two Variable (T) Independent Unequal Variance
Column 1 Observations: 6
Column 1 Sample Mean: 46.233333
Column 1 Sample Standard Deviation: 13.619202
Column 2 Observations: 6
Column 2 Sample Mean: 13.066667
Column 2 Sample Standard Deviation: 4.943548
Sample Mean Difference: 33.166667
t-Statistic: 5.607240
Hypothesized Mean: 0.000000

p-Value Left Tailed: 0.999314
not significant at any of the following significance levels: 1%, 5%, and 10%
not rejected
not significantly less than the hypothesized mean difference.

p-Value Right Tailed: 0.000686
significant at 1%, 5% and 10%
rejected
significantly greater than the hypothesized mean difference.

p-Value Two Tailed: 0.001372
significant at 1%, 5% and 10%
rejected
significantly different than the hypothesized mean difference.

--------------------------------------------------------------------------------------
```

Nonparametric Two Variable Wilcoxon Signed Rank Test

```
Model Inputs:
VAR1; VAR2
NPS, MS


             Wilcoxon Signed-Rank (Two Var)
Observations: 18
W1 Statistic: 112.000000
W2 Statistic: 59.000000
Z Approximation: 2.340007
P-Value 1 Tail: 0.009642
P-Value 2 Tail: 0.019283

-------------------------------------------------------------------------------------

Model Inputs:
VAR1; VAR3
NPS, BS


             Wilcoxon Signed-Rank (Two Var)
Observations: 18
W1 Statistic: 125.000000
W2 Statistic: 46.000000
Z Approximation: 3.487935
P-Value 1 Tail: 0.000243
P-Value 2 Tail: 0.000487

-------------------------------------------------------------------------------------

Model Inputs:
VAR2; VAR3
MS, BS


             Wilcoxon Signed-Rank (Two Var)
Observations: 18
W1 Statistic: 114.000000
W2 Statistic: 57.000000
Z Approximation: 2.516611
P-Value 1 Tail: 0.005924
P-Value 2 Tail: 0.011849

-------------------------------------------------------------------------------------

Model Inputs:
VAR5; VAR6
NPS, MS


             Wilcoxon Signed-Rank (Two Var)
Observations: 18
W1 Statistic: 123.000000
W2 Statistic: 48.000000
Z Approximation: 3.311331
P-Value 1 Tail: 0.000464
P-Value 2 Tail: 0.000929

-------------------------------------------------------------------------------------
```

```
Model Inputs:
VAR5; VAR7
NPS, BS


             Wilcoxon Signed-Rank (Two Var)
Observations: 18
W1 Statistic: 126.000000
W2 Statistic: 45.000000
Z Approximation: 3.576237
P-Value 1 Tail: 0.000174
P-Value 2 Tail: 0.000349

--------------------------------------------------------------------------------------

Model Inputs:
VAR6; VAR7
MS, BS


             Wilcoxon Signed-Rank (Two Var)
Observations: 18
W1 Statistic: 122.000000
W2 Statistic: 49.000000
Z Approximation: 3.223029
P-Value 1 Tail: 0.000634
P-Value 2 Tail: 0.001268

--------------------------------------------------------------------------------------

Model Inputs:
VAR9; VAR10
NPS, MS


             Wilcoxon Signed-Rank (Two Var)
Observations: 18
W1 Statistic: 126.000000
W2 Statistic: 45.000000
Z Approximation: 3.576237
P-Value 1 Tail: 0.000174
P-Value 2 Tail: 0.000349

--------------------------------------------------------------------------------------

Model Inputs:
VAR9; VAR11
NPS, BS


             Wilcoxon Signed-Rank (Two Var)
Observations: 18
W1 Statistic: 126.000000
W2 Statistic: 45.000000
Z Approximation: 3.576237
P-Value 1 Tail: 0.000174
P-Value 2 Tail: 0.000349

--------------------------------------------------------------------------------------
```

```
Model Inputs:
VAR10; VAR11
MS, BS


             Wilcoxon Signed-Rank (Two Var)
Observations: 18
W1 Statistic: 126.000000
W2 Statistic: 45.000000
Z Approximation: 3.576237
P-Value 1 Tail: 0.000174
P-Value 2 Tail: 0.000349

--------------------------------------------------------------------------------------

Model Inputs:
VAR13; VAR14
NPS, MS


             Wilcoxon Signed-Rank (Two Var)
Observations: 18
W1 Statistic: 123.000000
W2 Statistic: 48.000000
Z Approximation: 3.311331
P-Value 1 Tail: 0.000464
P-Value 2 Tail: 0.000929

--------------------------------------------------------------------------------------

Model Inputs:
VAR13; VAR15
NPS, BS


             Wilcoxon Signed-Rank (Two Var)
Observations: 18
W1 Statistic: 126.000000
W2 Statistic: 45.000000
Z Approximation: 3.576237
P-Value 1 Tail: 0.000174
P-Value 2 Tail: 0.000349

--------------------------------------------------------------------------------------

Model Inputs:
VAR14; VAR15
MS, BS


             Wilcoxon Signed-Rank (Two Var)
Observations: 18
W1 Statistic: 126.000000
W2 Statistic: 45.000000
Z Approximation: 3.576237
P-Value 1 Tail: 0.000174
P-Value 2 Tail: 0.000349

--------------------------------------------------------------------------------------
```

```
Model Inputs:
VAR17; VAR18
NPS, MS


             Wilcoxon Signed-Rank (Two Var)
Observations: 12
W1 Statistic: 46.000000
W2 Statistic: 32.000000
Z Approximation: 1.120897
P-Value 1 Tail: 0.131166
P-Value 2 Tail: 0.262332

--------------------------------------------------------------------------------------

Model Inputs:
VAR17; VAR19
NPS, BS


             Wilcoxon Signed-Rank (Two Var)
Observations: 12
W1 Statistic: 57.000000
W2 Statistic: 21.000000
Z Approximation: 2.882307
P-Value 1 Tail: 0.001974
P-Value 2 Tail: 0.003948

--------------------------------------------------------------------------------------

Model Inputs:
VAR18; VAR19
MS, BS


             Wilcoxon Signed-Rank (Two Var)
Observations: 12
W1 Statistic: 57.000000
W2 Statistic: 21.000000
Z Approximation: 2.882307
P-Value 1 Tail: 0.001974
P-Value 2 Tail: 0.003948

--------------------------------------------------------------------------------------
```

Basic Econometrics Linear Regression Models

```
Model Inputs:
VAR22
VAR25
NPS
Years

Regression Results

OVERALL FIT
Multiple R             0.87975      Maximum Log Likelihood                 3.84566
R-Square               0.77395      Akaike Info Criterion (AIC)           -0.73826
Adjusted R-Square      0.69861      Bayes Schwarz Criterion (BSC)         -0.89449
Standard Error         0.10541      Hannan-Quinn Criterion (HQC)          -1.15756
Observations                 5


                   Coeff  Std. Error      T-stat     P-value    Lower 5%   Upper 95%
Intercept        1.18623     0.12062     9.83458     0.00223     0.80237     1.57009
VAR X1          -0.17640     0.05504    -3.20495     0.04915    -0.35157    -0.00124


ANOVA
                      DF              SS            MS             F       p-Value
Regression             1            0.11          0.11      10.27168       0.04915
Residual               3            0.03          0.01
Total                  4            0.15


Hypothesis Test
  Critical F-statistic (99% confidence with DFR1 and DFR2): 34.116222
  Critical F-statistic (95% confidence with DFR1 and DFR2): 10.127964
  Critical F-statistic (90% confidence with DFR1 and DFR2): 5.538319


Forecasting
        Period        Actual (Y)      Forecast (F)         Error (E)
             1            0.9931            1.0640           -0.0709
             2            0.9892            0.9417            0.0475
             3            0.9038            0.7801            0.1237
             4            0.7117            0.7085            0.0032
             5            0.5542            0.6578           -0.1036

--------------------------------------------------------------------------------------
```

```
Model Inputs:
VAR23
VAR25
MS
Years

Regression Results

OVERALL FIT
Multiple R             0.96003     Maximum Log Likelihood            5.35519
R-Square               0.92165     Akaike Info Criterion (AIC)      -1.34208
Adjusted R-Square      0.89553     Bayes Schwarz Criterion (BSC)    -1.49830
Standard Error         0.07227     Hannan-Quinn Criterion (HQC)     -1.76137
Observations                 5


                   Coeff  Std. Error      T-stat     P-value    Lower 5%   Upper 95%
Intercept        1.19468     0.08270    14.44552     0.00072     0.93149     1.45788
VAR X1          -0.22419     0.03774    -5.94048     0.00954    -0.34429    -0.10409


ANOVA
                      DF              SS            MS             F       p-Value
Regression             1            0.18          0.18      35.28935       0.00954
Residual               3            0.02          0.01
Total                  4            0.20


Hypothesis Test
  Critical F-statistic (99% confidence with DFR1 and DFR2): 34.116222
  Critical F-statistic (95% confidence with DFR1 and DFR2): 10.127964
  Critical F-statistic (90% confidence with DFR1 and DFR2): 5.538319


Forecasting
        Period        Actual (Y)      Forecast (F)         Error (E)
             1            0.9771            1.0393           -0.0622
             2            0.9453            0.8839            0.0614
             3            0.7442            0.6785            0.0657
             4            0.5834            0.5876           -0.0042
             5            0.4623            0.5231           -0.0608

--------------------------------------------------------------------------------------

Model Inputs:
VAR24
VAR25
BS
Years

Regression Results

OVERALL FIT
Multiple R             0.98559     Maximum Log Likelihood            5.24511
R-Square               0.97140     Akaike Info Criterion (AIC)      -1.29804
Adjusted R-Square      0.96186     Bayes Schwarz Criterion (BSC)    -1.45427
Standard Error         0.07429     Hannan-Quinn Criterion (HQC)     -1.71734
Observations                 5
```

```
                   Coeff   Std. Error       T-stat      P-value    Lower 5%    Upper 95%
Intercept        1.29396      0.08501     15.22126      0.00062     1.02342      1.56450
VAR X1          -0.39155      0.03879    -10.09363      0.00207    -0.51500     -0.26810


ANOVA
                     DF              SS             MS             F        p-Value
Regression            1            0.56           0.56     101.88146        0.00207
Residual              3            0.02           0.01
Total                 4            0.58


Hypothesis Test
  Critical F-statistic (99% confidence with DFR1 and DFR2): 34.116222
  Critical F-statistic (95% confidence with DFR1 and DFR2): 10.127964
  Critical F-statistic (90% confidence with DFR1 and DFR2): 5.538319


Forecasting
        Period         Actual (Y)       Forecast (F)         Error (E)
             1             0.9578             1.0226           -0.0648
             2             0.8526             0.7512            0.1014
             3             0.3904             0.3924           -0.0020
             4             0.1892             0.2336           -0.0444
             5             0.1307             0.1210            0.0097

-------------------------------------------------------------------------------------
```

Basic Econometrics Nonlinear Regression Models

```
Model Inputs:
VAR22
VAR25
NPS
Years

Regression Results

OVERALL FIT
Multiple R                0.97439      Maximum Log Likelihood                 6.84033
R-Square                  0.94943      Akaike Info Criterion (AIC)           -1.93613
Adjusted R-Square         0.93257      Bayes Schwarz Criterion (BSC)         -2.09236
Standard Error            0.04986      Hannan-Quinn Criterion (HQC)          -2.35543
Observations                    5


                   Coeff   Std. Error       T-stat      P-value    Lower 5%    Upper 95%
Intercept        1.08494      0.04059     26.72951      0.00011     0.95577      1.21412
VAR X1          -0.02496      0.00333     -7.50475      0.00490    -0.03554     -0.01437


ANOVA
                     DF              SS             MS             F        p-Value
Regression            1            0.14           0.14      56.32134        0.00490
Residual              3            0.01           0.00
Total                 4            0.15
```

```
Hypothesis Test
  Critical F-statistic (99% confidence with DFR1 and DFR2): 34.116222
  Critical F-statistic (95% confidence with DFR1 and DFR2): 10.127964
  Critical F-statistic (90% confidence with DFR1 and DFR2): 5.538319


Forecasting
        Period        Actual (Y)      Forecast (F)         Error (E)
             1            0.9931            1.0350           -0.0419
             2            0.9892            0.9851            0.0041
             3            0.9038            0.8354            0.0684
             4            0.7117            0.7106            0.0011
             5            0.5542            0.5858           -0.0316

--------------------------------------------------------------------------------------

Model Inputs:
VAR23
VAR25
MS
Years

Regression Results

OVERALL FIT
Multiple R              0.99791      Maximum Log Likelihood               11.21530
R-Square                0.99582      Akaike Info Criterion (AIC)          -3.68612
Adjusted R-Square       0.99442      Bayes Schwarz Criterion (BSC)        -3.84234
Standard Error          0.01670      Hannan-Quinn Criterion (HQC)         -4.10541
Observations                  5


                   Coeff  Std. Error      T-stat     P-value    Lower 5%   Upper 95%
Intercept        1.04606     0.01360    76.93884     0.00000     1.00279     1.08933
VAR X1          -0.02976     0.00111   -26.72278     0.00011    -0.03331    -0.02622


ANOVA
                      DF              SS            MS              F       p-Value
Regression             1            0.20          0.20      714.10708       0.00011
Residual               3            0.00          0.00
Total                  4            0.20


Hypothesis Test
  Critical F-statistic (99% confidence with DFR1 and DFR2): 34.116222
  Critical F-statistic (95% confidence with DFR1 and DFR2): 10.127964
  Critical F-statistic (90% confidence with DFR1 and DFR2): 5.538319


Forecasting
        Period        Actual (Y)      Forecast (F)         Error (E)
             1            0.9771            0.9865           -0.0094
             2            0.9453            0.9270            0.0183
             3            0.7442            0.7484           -0.0042
             4            0.5834            0.5996           -0.0162
             5            0.4623            0.4508            0.0115

--------------------------------------------------------------------------------------
```

```
Model Inputs:
VAR24
VAR25
BS
Years

Regression Results

OVERALL FIT
Multiple R             0.96688      Maximum Log Likelihood                3.59924
R-Square               0.93486      Akaike Info Criterion (AIC)          -0.63969
Adjusted R-Square      0.91315      Bayes Schwarz Criterion (BSC)        -0.79592
Standard Error         0.11210      Hannan-Quinn Criterion (HQC)         -1.05899
Observations                 5

                   Coeff  Std. Error      T-stat     P-value    Lower 5%   Upper 95%
Intercept        1.00457     0.09127    11.00698     0.00161     0.71412     1.29502
VAR X1          -0.04906     0.00748    -6.56184     0.00720    -0.07286    -0.02527


ANOVA
                      DF              SS            MS             F       p-Value
Regression             1            0.54          0.54      43.05769       0.00720
Residual               3            0.04          0.01
Total                  4            0.58


Hypothesis Test
  Critical F-statistic (99% confidence with DFR1 and DFR2): 34.116222
  Critical F-statistic (95% confidence with DFR1 and DFR2): 10.127964
  Critical F-statistic (90% confidence with DFR1 and DFR2): 5.538319


Forecasting
        Period        Actual (Y)      Forecast (F)         Error (E)
             1            0.9578            0.9064            0.0514
             2            0.8526            0.8083            0.0443
             3            0.3904            0.5140           -0.1236
             4            0.1892            0.2686           -0.0794
             5            0.1307            0.0233            0.1074

--------------------------------------------------------------------------------------
```

Basic Econometrics Regression Through the Origin Models

```
Model Inputs:
VAR22
VAR25
NPS
Years

Multiple Regression Through the Origin


OVERALL FIT
Multiple R             0.70991    Maximum Log Likelihood             -27.54322
R-Square               0.50397    Akaike Info Criterion (AIC)         11.41729
Adjusted R-Square      0.37996    Bayes Schwarz Criterion (BSC)       11.33917
Standard Error        66.77153    Hannan-Quinn Criterion (HQC)        11.20764
Observations                 5





ANOVA
            DF          SS          MS          F           p-Value
Regression  1           18118.98    18118.98    4.06398     0.11403
Residual    4           17833.75    4458.44
Total       5           35952.73


            Coeff       Std. Error  T-stat      P-value     Lower 5%    Upper 95%
VAR X1      4.93161     2.44632     2.01593     0.11403     -1.86046    11.72368

--------------------------------------------------------------------------------------

Model Inputs:
VAR23
VAR25
MS
Years

Multiple Regression Through the Origin


OVERALL FIT
Multiple R             0.66428    Maximum Log Likelihood             -27.35152
R-Square               0.44127    Akaike Info Criterion (AIC)         11.34061
Adjusted R-Square      0.30158    Bayes Schwarz Criterion (BSC)       11.26250
Standard Error        64.26001    Hannan-Quinn Criterion (HQC)        11.13096
Observations                 5


ANOVA
            DF          SS          MS          F           p-Value
Regression  1           13044.88    13044.88    3.15906     0.15014
Residual    4           16517.39    4129.35
Total       5           29562.27


            Coeff       Std. Error  T-stat      P-value     Lower 5%    Upper 95%
VAR X1      4.18448     2.35430     1.77738     0.15014     -2.35211    10.72108

--------------------------------------------------------------------------------------
```

```
Model Inputs:
VAR24
VAR25
BS
Years

Multiple Regression Through the Origin


OVERALL FIT
Multiple R           0.39552    Maximum Log Likelihood           -27.20907
R-Square             0.15644    Akaike Info Criterion (AIC)       11.28363
Adjusted R-Square   -0.05446    Bayes Schwarz Criterion (BSC)     11.20552
Standard Error      62.45509    Hannan-Quinn Criterion (HQC)      11.07398
Observations               5





ANOVA
            DF          SS          MS          F           p-Value
Regression  1           2893.44     2893.44     0.74179     0.43766
Residual    4           15602.55    3900.64
Total       5           18495.99


            Coeff       Std. Error  T-stat      P-value     Lower 5%    Upper 95%
VAR X1      1.97074     2.28818     0.86127     0.43766     -4.38226    8.32374

-------------------------------------------------------------------------------------
```

```
Multiple Regression Functional Form Tests
Model Inputs: VAR22, VAR25


Func. Form       R-Squared        Adj. R-Squared   Ind. P-Value     Akaike           Bayes Schwarz
Linear           0.949428         0.932571         0.004902         17.511479        16.730355
Linear Log       0.773955         0.698606         0.049146         24.998164        24.217040
Reciprocal       0.541279         0.388372         0.156468         28.536695        27.755571
Quadratic        0.990305         0.980609         0.100953         9.252756         8.471631
Log Linear       0.925346         0.900461         0.008859        -23.868894       -24.650018
Log Reciprocal   0.498235         0.330979         0.182816        -14.342562       -15.123686
Log Quadratic    0.995432         0.990865         0.031075        -37.838149       -38.619273
Double Log       0.730667         0.640890         0.064958        -17.453488       -18.234612
Logistic         0.892158         0.856211         0.015547        -64.761916       -65.543041




Linear Regression: Y on X
Multiple R            0.97439    Akaike Info Criterion (AIC)        17.51148
R-Square              0.94943    AIC Correction (AICC)              41.51148
Adjusted R-Square     0.93257    Bayes Schwarz Criterion (BSC)      16.73036
Standard Error        4.98563    Augmented AIC                      31.70086
Observations                5    Augmented BSC                      30.91974


ANOVA
                 DF               SS               MS               F                p-Value
Regression       1                1399.95          1399.95          56.32134         0.00490
Residual         3                74.57            24.86
Total            4                1474.52


                 Coeff            Std. Error       T-stat           P-value
Intercept        108.49417        4.05897          26.72951         0.00011
VAR X1           -2.49551         0.33252          -7.50475         0.00490




Linear Log Regression: Y on LN(X)
Multiple R            0.87975    Akaike Info Criterion (AIC)        24.99816
R-Square              0.77395    AIC Correction (AICC)              48.99816
Adjusted R-Square     0.69861    Bayes Schwarz Criterion (BSC)      24.21704
Standard Error       10.54054    Augmented AIC                      39.18755
Observations                5    Augmented BSC                      38.40642


ANOVA
                 DF               SS               MS               F                p-Value
Regression       1                1141.21          1141.21          10.27168         0.04915
Residual         3                333.31           111.10
Total            4                1474.52


                 Coeff            Std. Error       T-stat           P-value
Intercept        118.62322        12.06185         9.83458          0.00223
VAR X1           -17.64024        5.50407          -3.20495         0.04915


Reciprocal Regression: Y on 1/X
Multiple R            0.73572    Akaike Info Criterion (AIC)        28.53670
R-Square              0.54128    AIC Correction (AICC)              52.53670
Adjusted R-Square     0.38837    Bayes Schwarz Criterion (BSC)      27.75557
Standard Error       15.01549    Augmented AIC                      42.72608
Observations                5    Augmented BSC                      41.94496
```

```
ANOVA
                DF              SS              MS              F               p-Value
Regression      1               798.13          798.13          3.53993         0.15647
Residual        3               676.39          225.46
Total           4               1474.52


                Coeff           Std. Error      T-stat          P-value
Intercept       68.57390        10.20830        6.71746         0.00673
VAR X1          74.82463        39.76925        1.88147         0.15647



Quadratic Regression: Y on X and X*X
Multiple R           0.99514    Akaike Info Criterion (AIC)          9.25276
R-Square             0.99030    AIC Correction (AICC)               33.25276
Adjusted R-Square    0.98061    Bayes Schwarz Criterion (BSC)        8.47163
Standard Error       2.67359    Augmented AIC                       23.44214
Observations               5    Augmented BSC                       22.66102


ANOVA
                DF              SS              MS              F               p-Value
Regression      2               1460.23         730.11          102.14148       0.00970
Residual        2               14.30           7.15
Total           4               1474.52


                Coeff           Std. Error      T-stat          P-value
Intercept       100.95466       3.38810         29.79687        0.00112
VAR X1          -0.20451        0.80886         -0.25283        0.82401
VAR X2          -0.10623        0.03658         -2.90382        0.10095



Log Linear Regression: LN(Y) on X
Multiple R           0.96195    Akaike Info Criterion (AIC)        -23.86889
R-Square             0.92535    AIC Correction (AICC)                0.13111
Adjusted R-Square    0.90046    Bayes Schwarz Criterion (BSC)      -24.65002
Standard Error       0.07954    Augmented AIC                       -9.67951
Observations               5    Augmented BSC                      -10.46063


ANOVA
                DF              SS              MS              F               p-Value
Regression      1               0.24            0.24            37.18529        0.00886
Residual        3               0.02            0.01
Total           4               0.25


                Coeff           Std. Error      T-stat          P-value
Intercept       4.72529         0.06476         72.96929        0.00001
VAR X1          -0.03235        0.00531         -6.09797        0.00886



Log Reciprocal Regression: LN(Y) on 1/X
Multiple R           0.70586    Akaike Info Criterion (AIC)        -14.34256
R-Square             0.49823    AIC Correction (AICC)                9.65744
Adjusted R-Square    0.33098    Bayes Schwarz Criterion (BSC)      -15.12369
Standard Error       0.20621    Augmented AIC                       -0.15318
Observations               5    Augmented BSC                       -0.93430
```

```
ANOVA
                DF              SS              MS              F               p-Value
Regression      1               0.13            0.13            2.97889         0.18282
Residual        3               0.13            0.04
Total           4               0.25


                Coeff           Std. Error      T-stat          P-value
Intercept       4.21307         0.14019         30.05162        0.00008
VAR X1          0.94265         0.54617         1.72595         0.18282



Log Quadratic Regression: LN(Y) on X and X*X
Multiple R           0.99771    Akaike Info Criterion (AIC)        -37.83815
R-Square             0.99543    AIC Correction (AICC)              -13.83815
Adjusted R-Square    0.99086    Bayes Schwarz Criterion (BSC)      -38.61927
Standard Error       0.02410    Augmented AIC                      -23.64876
Observations               5    Augmented BSC                      -24.42989


ANOVA
                DF              SS              MS              F               p-Value
Regression      2               0.25            0.13            217.92733       0.00457
Residual        2               0.00            0.00
Total           4               0.25


                Coeff           Std. Error      T-stat          P-value
Intercept       4.59566         0.03054         150.49567       0.00004
VAR X1          0.00704         0.00729         0.96586         0.43602
VAR X2          -0.00183        0.00033         -5.53965        0.03108



Double Log Regression: LN(Y) on LN(X)
Multiple R           0.85479    Akaike Info Criterion (AIC)        -17.45349
R-Square             0.73067    AIC Correction (AICC)                6.54651
Adjusted R-Square    0.64089    Bayes Schwarz Criterion (BSC)      -18.23461
Standard Error       0.15108    Augmented AIC                       -3.26410
Observations               5    Augmented BSC                       -4.04523


ANOVA
                DF              SS              MS              F               p-Value
Regression      1               0.19            0.19            8.13864         0.06496
Residual        3               0.07            0.02
Total           4               0.25


                Coeff           Std. Error      T-stat          P-value
Intercept       4.84931         0.17289         28.04903        0.00010
VAR X1          -0.22506        0.07889         -2.85283        0.06496



Logistic Regression: Y/(1-Y) on X
Multiple R           0.94454    Akaike Info Criterion (AIC)        -64.76192
R-Square             0.89216    AIC Correction (AICC)              -40.76192
Adjusted R-Square    0.85621    Bayes Schwarz Criterion (BSC)      -65.54304
Standard Error       0.00133    Augmented AIC                      -50.57253
Observations               5    Augmented BSC                      -51.35366



ANOVA
```

```
                DF              SS              MS              F               p-Value
Regression      1               0.00            0.00            24.81855        0.01555
Residual        3               0.00            0.00
Total           4               0.00


                Coeff           Std. Error      T-stat          P-value
Intercept       -1.00832        0.00108         -929.55033      0.00000
VAR X1          -0.00044        0.00009         -4.98182        0.01555


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Econometric Models of NON-NPS Graduate Degrees over the Years with Functional Forms Test

```
Multiple Regression Functional Form Tests
Model Inputs: VAR23 and VAR25


Func. Form      R-Squared       Adj. R-Squared  Ind. P-Value    Akaike          Bayes Schwarz
Linear          0.995817        0.994422        0.000115        6.574077        5.792952
Linear Log      0.921649        0.895532        0.009536        21.224344       20.443219
Reciprocal      0.723206        0.630942        0.067869        27.534729       26.753605
Quadratic       0.996421        0.992843        0.619749        5.793199        5.012075
Log Linear      0.992850        0.990466        0.000257       -33.225146      -34.006270
Log Reciprocal  0.660358        0.547144        0.094581       -13.921361      -14.702485
Log Quadratic   0.997969        0.995937        0.153883       -39.517550      -40.298674
Double Log      0.875585        0.834113        0.019371       -18.942685      -19.723809
Logistic        0.969872        0.959829        0.002240       -67.683699      -68.464823




Linear Regression: Y on X
Multiple R           0.99791    Akaike Info Criterion (AIC)          6.57408
R-Square             0.99582    AIC Correction (AICC)               30.57408
Adjusted R-Square    0.99442    Bayes Schwarz Criterion (BSC)        5.79295
Standard Error       1.66999    Augmented AIC                       20.76346
Observations               5    Augmented BSC                       19.98234


ANOVA
                DF              SS              MS              F               p-Value
Regression      1               1991.56         1991.56         714.10708       0.00011
Residual        3               8.37            2.79
Total           4               1999.93


                Coeff           Std. Error      T-stat          P-value
Intercept       104.60579       1.35960         76.93884        0.00000
VAR X1          -2.97645        0.11138         -26.72278       0.00011



Linear Log Regression: Y on LN(X)
Multiple R           0.96003    Akaike Info Criterion (AIC)         21.22434
R-Square             0.92165    AIC Correction (AICC)               45.22434
Adjusted R-Square    0.89553    Bayes Schwarz Criterion (BSC)       20.44322
Standard Error       7.22717    Augmented AIC                       35.41373
Observations               5    Augmented BSC                       34.63260


ANOVA
```

```
                DF              SS              MS              F               p-Value
Regression      1               1843.23         1843.23         35.28935        0.00954
Residual        3               156.70          52.23
Total           4               1999.93


                Coeff           Std. Error      T-stat          P-value
Intercept       119.46817       8.27026         14.44552        0.00072
VAR X1          -22.41871       3.77389         -5.94048        0.00954



Reciprocal Regression: Y on 1/X
Multiple R            0.85042    Akaike Info Criterion (AIC)         27.53473
R-Square              0.72321    AIC Correction (AICC)               51.53473
Adjusted R-Square     0.63094    Bayes Schwarz Criterion (BSC)       26.75360
Standard Error       13.58391    Augmented AIC                       41.72411
Observations                5    Augmented BSC                       40.94299


ANOVA
                DF              SS              MS              F               p-Value
Regression      1               1446.36         1446.36         7.83839         0.06787
Residual        3               553.57          184.52
Total           4               1999.93


                Coeff           Std. Error      T-stat          P-value
Intercept       54.77210        9.23504         5.93090         0.00958
VAR X1          100.72705       35.97763        2.79971         0.06787



Quadratic Regression: Y on X and X*X
Multiple R            0.99821    Akaike Info Criterion (AIC)          5.79320
R-Square              0.99642    AIC Correction (AICC)               29.79320
Adjusted R-Square     0.99284    Bayes Schwarz Criterion (BSC)        5.01207
Standard Error        1.89168    Augmented AIC                       19.98258
Observations                5    Augmented BSC                       19.20146


ANOVA
                DF              SS              MS              F               p-Value
Regression      2               1992.77         996.39          278.44022       0.00358
Residual        2               7.16            3.58
Total           4               1999.93


                Coeff           Std. Error      T-stat          P-value
Intercept       105.67393       2.39723         44.08174        0.00051
VAR X1          -3.30102        0.57231         -5.76793        0.02877
VAR X2          0.01505         0.02588         0.58143         0.61975



Log Linear Regression: LN(Y) on X
Multiple R            0.99642    Akaike Info Criterion (AIC)        -33.22515
R-Square              0.99285    AIC Correction (AICC)               -9.22515
Adjusted R-Square     0.99047    Bayes Schwarz Criterion (BSC)      -34.00627
Standard Error        0.03121    Augmented AIC                      -19.03576
Observations                5    Augmented BSC                      -19.81688



ANOVA
```

```
                DF              SS              MS              F               p-Value
Regression      1               0.41            0.41            416.57129       0.00026
Residual        3               0.00            0.00
Total           4               0.41



                Coeff           Std. Error      T-stat          P-value
Intercept       4.70143         0.02541         185.04524       0.00000
VAR X1          -0.04248        0.00208         -20.41008       0.00026



Log Reciprocal Regression: LN(Y) on 1/X
Multiple R           0.81262    Akaike Info Criterion (AIC)        -13.92136
R-Square             0.66036    AIC Correction (AICC)               10.07864
Adjusted R-Square    0.54714    Bayes Schwarz Criterion (BSC)      -14.70249
Standard Error       0.21508    Augmented AIC                        0.26802
Observations               5    Augmented BSC                       -0.51310

ANOVA
                DF              SS              MS              F               p-Value
Regression      1               0.27            0.27            5.83284         0.09458
Residual        3               0.14            0.05
Total           4               0.41

                Coeff           Std. Error      T-stat          P-value
Intercept       4.00212         0.14623         27.36950        0.00011
VAR X1          1.37581         0.56966         2.41513         0.09458



Log Quadratic Regression: LN(Y) on X and X*X
Multiple R           0.99898    Akaike Info Criterion (AIC)        -39.51755
R-Square             0.99797    AIC Correction (AICC)              -15.51755
Adjusted R-Square    0.99594    Bayes Schwarz Criterion (BSC)      -40.29867
Standard Error       0.02037    Augmented AIC                      -25.32816
Observations               5    Augmented BSC                      -26.10929

ANOVA
                DF              SS              MS              F               p-Value
Regression      2               0.41            0.20            491.30670       0.00203
Residual        2               0.00            0.00
Total           4               0.41

                Coeff           Std. Error      T-stat          P-value
Intercept       4.65701         0.02582         180.39277       0.00003
VAR X1          -0.02899        0.00616         -4.70301        0.04236
VAR X2          -0.00063        0.00028         -2.24503        0.15388



Double Log Regression: LN(Y) on LN(X)
Multiple R           0.93573    Akaike Info Criterion (AIC)        -18.94269
R-Square             0.87558    AIC Correction (AICC)                5.05731
Adjusted R-Square    0.83411    Bayes Schwarz Criterion (BSC)      -19.72381
Standard Error       0.13018    Augmented AIC                       -4.75330
Observations               5    Augmented BSC                       -5.53442


ANOVA
```

```
                DF              SS              MS              F               p-Value
Regression      1               0.36            0.36            21.11276        0.01937
Residual        3               0.05            0.02
Total           4               0.41


                Coeff           Std. Error      T-stat          P-value
Intercept       4.89816         0.14897         32.88103        0.00006
VAR X1          -0.31234        0.06798         -4.59486        0.01937



Logistic Regression: Y/(1-Y) on X
Multiple R           0.98482    Akaike Info Criterion (AIC)        -67.68370
R-Square             0.96987    AIC Correction (AICC)              -43.68370
Adjusted R-Square    0.95983    Bayes Schwarz Criterion (BSC)      -68.46482
Standard Error       0.00099    Augmented AIC                      -53.49431
Observations               5    Augmented BSC                      -54.27544


ANOVA
                DF              SS              MS              F               p-Value
Regression      1               0.00            0.00            96.57519        0.00224
Residual        3               0.00            0.00
Total           4               0.00


                Coeff           Std. Error      T-stat          P-value
Intercept       -1.00819        0.00081         -1244.81913     0.00000
VAR X1          -0.00065        0.00007         -9.82727        0.00224



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Econometric Models of NON-NPS Undergraduate Degrees over the Years with Functional Forms Test

```
Multiple Regression Functional Form Tests
Model Inputs: VAR24 and VAR25

Func. Form      R-Squared       Adj. R-Squared  Ind. P-Value    Akaike          Bayes Schwarz
Linear          0.934864        0.913152        0.007198        25.614223       24.833099
Linear Log      0.971396        0.961862        0.002071        21.499542       20.718418
Reciprocal      0.837661        0.783549        0.029244        30.180279       29.399155
Quadratic       0.994970        0.989940        0.039386        12.808816       12.027692
Log Linear      0.989549        0.986065        0.000455       -21.141257      -21.922381
Log Reciprocal  0.722389        0.629852        0.068191       -4.743626       -5.524751
Log Quadratic   0.991468        0.982936        0.571505       -22.155563      -22.936687
Double Log      0.920356        0.893808        0.009778       -10.986906      -11.768030
Logistic        0.953617        0.938156        0.004300       -47.134430      -47.915555



Linear Regression: Y on X
Multiple R           0.96688    Akaike Info Criterion (AIC)         25.61422
R-Square             0.93486    AIC Correction (AICC)               49.61422
Adjusted R-Square    0.91315    Bayes Schwarz Criterion (BSC)       24.83310
Standard Error      11.21032    Augmented AIC                       39.80361
Observations               5    Augmented BSC                       39.02248



ANOVA
```

```
                DF              SS              MS              F               p-Value
Regression      1               5411.12         5411.12         43.05769        0.00720
Residual        3               377.01          125.67
Total           4               5788.13


                Coeff           Std. Error      T-stat          P-value
Intercept       100.45725       9.12669         11.00698        0.00161
VAR X1          -4.90620        0.74769         -6.56184        0.00720



Linear Log Regression: Y on LN(X)
Multiple R           0.98559    Akaike Info Criterion (AIC)         21.49954
R-Square             0.97140    AIC Correction (AICC)               45.49954
Adjusted R-Square    0.96186    Bayes Schwarz Criterion (BSC)       20.71842
Standard Error       7.42882    Augmented AIC                       35.68893
Observations               5    Augmented BSC                       34.90780


ANOVA
                DF              SS              MS              F               p-Value
Regression      1               5622.57         5622.57         101.88146       0.00207
Residual        3               165.56          55.19
Total           4               5788.13


                Coeff           Std. Error      T-stat          P-value
Intercept       129.39614       8.50101         15.22126        0.00062
VAR X1          -39.15509       3.87919         -10.09363       0.00207



Reciprocal Regression: Y on 1/X
Multiple R           0.91524    Akaike Info Criterion (AIC)         30.18028
R-Square             0.83766    AIC Correction (AICC)               54.18028
Adjusted R-Square    0.78355    Bayes Schwarz Criterion (BSC)       29.39915
Standard Error      17.69781    Augmented AIC                       44.36966
Observations               5    Augmented BSC                       43.58854


ANOVA
                DF              SS              MS              F               p-Value
Regression      1               4848.49         4848.49         15.47990        0.02924
Residual        3               939.64          313.21
Total           4               5788.13


                Coeff           Std. Error      T-stat          P-value
Intercept       14.75919        12.03188        1.22667         0.30744
VAR X1          184.42143       46.87350        3.93445         0.02924



Quadratic Regression: Y on X and X*X
Multiple R           0.99748    Akaike Info Criterion (AIC)         12.80882
R-Square             0.99497    AIC Correction (AICC)               36.80882
Adjusted R-Square    0.98994    Bayes Schwarz Criterion (BSC)       12.02769
Standard Error       3.81533    Augmented AIC                       26.99820
Observations               5    Augmented BSC                       26.21708


ANOVA
                DF              SS              MS              F               p-Value
Regression      2               5759.02         2879.51         197.81281       0.00503
Residual        2               29.11           14.56
Total           4               5788.13
```

```
                Coeff           Std. Error      T-stat          P-value
Intercept       118.57097       4.83497         24.52364        0.00166
VAR X1          -10.41034       1.15428         -9.01887        0.01207
VAR X2          0.25523         0.05221         4.88872         0.03939



Log Linear Regression: LN(Y) on X
Multiple R           0.99476    Akaike Info Criterion (AIC)        -21.14126
R-Square             0.98955    AIC Correction (AICC)                2.85874
Adjusted R-Square    0.98607    Bayes Schwarz Criterion (BSC)      -21.92238
Standard Error       0.10448    Augmented AIC                       -6.95187
Observations               5    Augmented BSC                       -7.73300


ANOVA
                DF              SS              MS              F               p-Value
Regression      1               3.10            3.10            284.05446       0.00045
Residual        3               0.03            0.01
Total           4               3.13


                Coeff           Std. Error      T-stat          P-value
Intercept       4.83458         0.08506         56.83422        0.00001
VAR X1          -0.11745        0.00697         -16.85392       0.00045



Log Reciprocal Regression: LN(Y) on 1/X
Multiple R           0.84993    Akaike Info Criterion (AIC)         -4.74363
R-Square             0.72239    AIC Correction (AICC)               19.25637
Adjusted R-Square    0.62985    Bayes Schwarz Criterion (BSC)       -5.52475
Standard Error       0.53851    Augmented AIC                        9.44576
Observations               5    Augmented BSC                        8.66463


ANOVA
                DF              SS              MS              F               p-Value
Regression      1               2.26            2.26            7.80648         0.06819
Residual        3               0.87            0.29
Total           4               3.13


                Coeff           Std. Error      T-stat          P-value
Intercept       2.86614         0.36611         7.82871         0.00434
VAR X1          3.98501         1.42627         2.79401         0.06819



Log Quadratic Regression: LN(Y) on X and X*X
Multiple R           0.99572    Akaike Info Criterion (AIC)        -22.15556
R-Square             0.99147    AIC Correction (AICC)                1.84444
Adjusted R-Square    0.98294    Bayes Schwarz Criterion (BSC)      -22.93669
Standard Error       0.11562    Augmented AIC                       -7.96618
Observations               5    Augmented BSC                       -8.74730


ANOVA
                DF              SS              MS              F               p-Value
Regression      2               3.11            1.55            116.20456       0.00853
Residual        2               0.03            0.01
Total           4               3.13




                Coeff           Std. Error      T-stat          P-value
```

```
Intercept       4.90988         0.14652         33.50899        0.00089
VAR X1          -0.14033        0.03498         -4.01176        0.05688
VAR X2          0.00106         0.00158         0.67067         0.57150
```

```
Double Log Regression: LN(Y) on LN(X)
Multiple R           0.95935    Akaike Info Criterion (AIC)        -10.98691
R-Square             0.92036    AIC Correction (AICC)               13.01309
Adjusted R-Square    0.89381    Bayes Schwarz Criterion (BSC)      -11.76803
Standard Error       0.28844    Augmented AIC                        3.20248
Observations               5    Augmented BSC                        2.42136
```

```
ANOVA
                DF              SS              MS              F               p-Value
Regression      1               2.88            2.88            34.66765        0.00978
Residual        3               0.25            0.08
Total           4               3.13

                Coeff           Std. Error      T-stat          P-value
Intercept       5.42543         0.33007         16.43736        0.00049
VAR X1          -0.88682        0.15062         -5.88792        0.00978
```

```
Logistic Regression: Y/(1-Y) on X
Multiple R           0.97653    Akaike Info Criterion (AIC)        -47.13443
R-Square             0.95362    AIC Correction (AICC)              -23.13443
Adjusted R-Square    0.93816    Bayes Schwarz Criterion (BSC)      -47.91555
Standard Error       0.00777    Augmented AIC                      -32.94505
Observations               5    Augmented BSC                      -33.72617

ANOVA
                DF              SS              MS              F               p-Value
Regression      1               0.00            0.00            61.67909        0.00430
Residual        3               0.00            0.00
Total           4               0.00

                Coeff           Std. Error      T-stat          P-value
Intercept       -0.99598        0.00632         -157.53309      0.00000
VAR X1          -0.00407        0.00052         -7.85360        0.00430
```

XIII. APPENDIX III: Analytical Results for DAU Surveys

SUPERVISOR'S POINT OF VIEW

- VAR1: On a scale of 0% (not at all) to 100% (extremely critical), how critical is applying the content of this training to the employee's job success?
- VAR2: This training has improved the employee's job performance.
- VAR3: Given all factors, including this training, estimate how much this employee's job performance related to the course subject matter has improved since the training.
- VAR4: This training was a worthwhile investment for my organization.
- VAR5: I set expectations with this employee for this learning prior to their attending/participating in training.
- VAR6: This employee has set specific goals for using this training to do his/her job.
- VAR7: What percent of this employee's total work time do you feel he/she spends on tasks that require the knowledge/skills presented in this training?
- VAR8: What percent of new knowledge and skills learned from this training did you observe being applied by the employee to his/her job?
- VAR9: This training was a worthwhile investment in the employee's career development.
- VAR10: After training, this employee and I discussed how he/she will use the learning on his/her job.
- VAR11: I feel this employee has learned new knowledge or skills from this training.
- VAR12: Based on your response to the prior question, estimate how much of the improvement was a direct result of this training. For example, if you feel that half of the improvement was a direct result of the training, enter 50% here.
- VAR13: The employee has been able to successfully apply the knowledge/skills learned in this class to his/her job.

Inter Class Correlation for Inter-rater Reliability Test

Model Inputs: VAR1; VAR3; VAR7; VAR8; VAR12

	DF	Sums of Squares	Mean Square	F-Stat	p-Value
Rows	144	41.86	0.29	12.29797	**0.00000**
Columns	4	2.43	0.61	25.65703	**0.00000**
Error	576	13.61	0.02		
Total	724	57.90			

Interclass Correlation **0.65884**

A high ICC indicates a high level of reliability vs. low correlations mean low reliability and low consistency.

The null hypothesis indicates that there is zero statistical reliability among the five survey questions requiring percentage responses.

Conclusion: There is statistical reliability among the percentage responses. This means that the survey responses of the 145 supervisors who sent their employees for training were statistically reliable. We can conclude that the responses to the survey are valid and trustworthy, rather than being completed haphazardly.

Inter Class Correlation for Inter-rater Reliability Test

```
Model Inputs: VAR2; VAR4; VAR5; VAR6; VAR9; VAR10; VAR11; VAR13


              DF        Sums of Squares    Mean Square        F-Stat         p-Value
Rows          144       1325.15            9.20               16.20281       0.00000
Columns       7         67.26              9.61               16.91674       0.00000
Error         1008      572.49             0.57
Total         1159      1964.90

Interclass Correlation  0.63132
```

A high ICC indicates a high level of reliability vs. low correlations mean low reliability and low consistency.

The null hypothesis assumes zero statistical reliability among the eight Likert scale variables.

Conclusion: There is statistical reliability among the responses. This means that the survey responses of the 145 supervisors who sent their employees for training were statistically valid and reliable. We can conclude that the responses to the survey are valid and trustworthy, rather than being completed haphazardly.

Guttman's Lambda & Internal Consistency and Reliability Test

```
Model Inputs: VAR1; VAR3; VAR7; VAR8; VAR12


Covariance                           0.32607
Variance of Total                    1.45337
Guttman's Lambda                     0.89743

Split Half Approach

Correlation Coefficient              0.91731
Spearman-Brown Correction            0.95687

Odd-Even Split Approach

Correlation Coefficient              0.86427
Spearman-Brown Correction            0.92719
```

High correlations and lambda scores mean high reliability and high consistency.

The null hypothesis states that there is zero statistical consistency among the five percentage variables.

Conclusion: There is statistical consistency among the survey questions with percentage responses. We can conclude that the survey data are statistically valid and can be trusted.

Guttman's Lambda & Internal Consistency and Reliability Test

```
Model Inputs: VAR2; VAR4; VAR5; VAR6; VAR9; VAR10; VAR11; VAR13

Covariance                          17.64751
Variance of Total                   73.61925
Guttman's Lambda                     0.95885
```

```
Split Half Approach

Correlation Coefficient              0.90279
Spearman-Brown Correction            0.94891

Odd-Even Split Approach

Correlation Coefficient              0.92378
Spearman-Brown Correction            0.96038

Low correlations and lambda scores mean low reliability and low consistency.
```

The null hypothesis tested is such that there is zero statistical consistency among the eight Likert scale variables.

Conclusion: There is statistical consistency and reliability among the survey questions requiring percentage responses. We can conclude that the survey data are statistically valid and can be trusted.

One Variable T-Test for Means

```
Model Inputs: VAR4
Observations: 145
Hypothesized Mean: 3.500000
Sample Mean: 5.813793
Standard Deviation (Sample): 1.201771
t-Statistic: 23.183922
p-Value Right Tailed: 0.000000
p-Value Two Tailed: 0.000000
```

The null hypothesis tested is such that the average supervisor's view of the return on investment for training is zero or negative.

Conclusion: We find statistical significance at an alpha of 1%, indicating that, on average, supervisors view that the ROI is statistically significantly greater than zero (mid-point of a Likert scale).

One Way ANOVA with Randomized Multiple Treatments

```
Model Inputs: VAR1; VAR3; VAR7; VAR8; VAR12

                      DF           Sums of Sq.    Mean Square   F Stat    p-Value
Between Groups         4              2.43           0.61       7.8712     0.0000
Within Groups        720             55.47           0.08
Total                724             57.90           0.08

F Critical @ 0.10                 1.952683
F Critical @ 0.05                 2.384302
F Critical @ 0.01                 3.345289
```

The null hypothesis tested is such that all the survey questions requiring percentage inputs are statistically equivalent.

Conclusion: The hypothesis is rejected, indicating that at least one of the variables is different from the rest. To identify the differences, additional analysis is required.

One Way ANOVA with Randomized Multiple Treatments

```
Model Inputs: VAR2; VAR4; VAR5; VAR6; VAR9; VAR10; VAR11; VAR13
                    DF          Sums of Sq.    Mean Square    F Stat    p-Value
Between Groups        7             67.26           9.61      5.8327     0.0000
Within Groups      1152           1897.64           1.65
Total              1159           1964.90           1.70

F Critical @ 0.10                1.721954
F Critical @ 0.05                2.017514
F Critical @ 0.01                2.654811
```

The null hypothesis tested is such that all the survey questions requiring Likert scale inputs are statistically equivalent.

Conclusion: The hypothesis is rejected, indicating that at least one of the variables is different from the rest. To identify the differences, additional analysis is required.

Kruskal-Wallis Test

```
Model Inputs: VAR1; VAR3; VAR7; VAR8; VAR12

H Statistic: 31.849544
p-Value: 0.000002
H Critical at 1%: 13.276704
H Critical at 5%: 9.487729
H Critical at 10%: 7.779440
The population medians are statistically not equal at 1%, 5%, or 10% significance.
```

Kruskal-Wallis Test

```
Model Inputs: VAR2; VAR4; VAR5; VAR6; VAR9; VAR10; VAR11; VAR13

H Statistic: 46.309683
p-Value: 0.000000
H Critical at 1%: 18.475307
H Critical at 5%: 14.067140
H Critical at 10%: 12.017037
The population medians are statistically not equal at 1%, 5%, or 10% significance.
```

Two Variable (T) Independent Equal Variance

```
Model Inputs: VAR4; VAR9
Column 1 Observations: 145          Column 2 Observations: 145
Column 1 Sample Mean: 5.813793      Column 1 Sample Standard Deviation: 1.201771
Column 2 Sample Mean: 5.848276      Column 2 Sample Standard Deviation: 1.138536
Sample Mean Difference: -0.034483
t-Statistic: -0.250824
Hypothesized Mean: 0.000000
p-Value Two Tailed: 0.802129
```

The null hypothesis tested is such that the ROI for the organization is statistically equal to the ROI for the employee's career development.

Conclusion: This is found to be not statistically significant, indicating that organizations view the ROI to an employee as being the same as the ROI to the organization.

Two Variable (T) Independent Equal Variance

```
Model Inputs: VAR4; VAR2
Column 1 Observations: 145              Column 2 Observations: 145
Column 1 Sample Mean: 5.813793          Column 1 Sample Standard Deviation: 1.201771
Column 2 Sample Mean: 5.413793          Column 2 Sample Standard Deviation: 1.250479
Sample Mean Difference: 0.400000
t-Statistic: 2.777210
Hypothesized Mean: 0.000000
p-Value Two Tailed: 0.005843
```

The null hypothesis tested is such that the ROI for the organization is statistically equal to the improvement of an employee’s job performance.

Conclusion: This is found to be statistically significantly different, indicating that the organization’s view of the ROI of a training initiative goes beyond its impact on an employee’s job performance.

Two Variable (T) Independent Equal Variance

```
Model Inputs: VAR2; VAR9
Column 1 Observations: 145              Column 2 Observations: 145
Column 1 Sample Mean: 5.413793          Column 1 Sample Standard Deviation: 1.250479
Column 2 Sample Mean: 5.848276          Column 2 Sample Standard Deviation: 1.138536
Sample Mean Difference: -0.434483
t-Statistic: -3.093687
Hypothesized Mean: 0.000000
p-Value Two Tailed: 0.002171
```

The null hypothesis tested is such that the ROI for the employee’s career development is statistically equal to the improvement of an employee’s job performance.

Conclusion: This is found to be statistically significantly different, indicating that the organization’s view of the ROI of a training initiative goes beyond its impact on an employee’s job performance. This might mean that the value of training is not entirely quantifiable or immediately actionable, and that some value might be intrinsic, unmeasurable, and subjective.

Two Variable (T) Independent Equal Variance

```
Model Inputs: VAR4; VAR11
Column 1 Observations: 145              Column 2 Observations: 145
Column 1 Sample Mean: 5.813793          Column 1 Sample Standard Deviation: 1.201771
Column 2 Sample Mean: 5.806897          Column 2 Sample Standard Deviation: 1.088427
Sample Mean Difference: 0.006897
t-Statistic: 0.051218
Hypothesized Mean: 0.000000
p-Value Two Tailed: 0.959187
```

The null hypothesis tested is such that the ROI for the organization is statistically equal to the amount of new knowledge or skills obtained from the training course.

Conclusion: This is found to be not statistically significant, indicating that organizations view the ROI to the organization as more than a simple summation of actual enumerable skills or new knowledge learned.

Two Variable (T) Independent Equal Variance

```
Model Inputs: VAR4; VAR13
Column 1 Observations: 145       Column 2 Observations: 145
Column 1 Sample Mean: 5.813793          Column 1 Sample Standard Deviation: 1.201771
Column 2 Sample Mean: 5.648276          Column 2 Sample Standard Deviation: 1.133646
Sample Mean Difference: 0.165517
t-Statistic: 1.206405
Hypothesized Mean: 0.000000
p-Value Two Tailed: 0.228651
```

The null hypothesis tested is such that the ROI for the organization is statistically equal to the amount of application of the knowledge or skills learned in the class to his/her job.

Conclusion: This is found to be not statistically significant, indicating that organizations view the ROI to the organization as being more than a simple summation of actual enumerable applications of specific knowledge or skill set on the job.

Nonparametric Mann-Whitney Test
(Two Independent Samples)

```
Model Inputs: VAR4; VAR9

                    Sample 1            Sample 2
Count               145                 145
Median              6.00                6.00
Rank Sum            21031.00            21164.00
U Values            10579.00            10446.00


Wilcoxon W          21031.00
U-Stat              10446.00
Mean                10512.50
Std Dev             714.04219
Z-Score             0.09243
P-value (One Tail)  0.46318
P-value (Two Tail)  0.92636
* Adjusted for Ties
```

Null hypothesis: There is zero difference between the two variables.

Nonparametric Mann-Whitney Test
(Two Independent Samples)

```
Model Inputs: VAR4; VAR2

                    Sample 1            Sample 2
Count               145                 145
Median              6.00                6.00
Rank Sum            23135.50            19059.50
U Values            8474.50             12550.50


Wilcoxon W          23135.50
U-Stat              8474.50
Mean                10512.50
Std Dev             714.04219
Z-Score             2.85347
P-value (One Tail)  0.00216
```

```
P-value (Two Tail)  0.00432
* Adjusted for Ties

Null hypothesis: There is zero difference between the two variables.

--------------------------------------------------------------------------------------
```

Nonparametric Mann-Whitney Test

```
(Two Independent Samples)

Model Inputs: VAR2; VAR9

                    Sample 1            Sample 2
Count               145                 145
Median              6.00                6.00
Rank Sum            18961.50            23233.50
U Values            12648.50            8376.50


Wilcoxon W          18961.50
U-Stat              8376.50
Mean                10512.50
Std Dev             714.04219
Z-Score             2.99072
P-value (One Tail)  0.00139
P-value (Two Tail)  0.00278
* Adjusted for Ties

Null hypothesis: There is zero difference between the two variables.

--------------------------------------------------------------------------------------
```

Nonparametric Mann-Whitney Test

```
(Two Independent Samples)

Model Inputs: VAR4; VAR11

                    Sample 1            Sample 2
Count               145                 145
Median              6.00                6.00
Rank Sum            21358.50            20836.50
U Values            10251.50            10773.50


Wilcoxon W          21358.50
U-Stat              10251.50
Mean                10512.50
Std Dev             714.04219
Z-Score             0.36482
P-value (One Tail)  0.35762
P-value (Two Tail)  0.71524
* Adjusted for Ties

Null hypothesis: There is zero difference between the two variables.

--------------------------------------------------------------------------------------
```

Nonparametric Mann-Whitney Test

```
(Two Independent Samples)

Model Inputs: VAR4; VAR13


                    Sample 1            Sample 2
Count               145                 145
Median              6.00                6.00
Rank Sum            22160.50            20034.50
U Values            9449.50             11575.50


Wilcoxon W          22160.50
U-Stat              9449.50
Mean                10512.50
Std Dev             714.04219
Z-Score             1.48801
P-value (One Tail)  0.06837
P-value (Two Tail)  0.13675
* Adjusted for Ties

Null hypothesis: There is zero difference between the two variables.

--------------------------------------------------------------------------------------
```

Basic Econometrics and Regression

```
Model Inputs: VAR4 vs. VAR1; VAR2; VAR3; VAR5; VAR6; VAR7; VAR8; VAR9; VAR10; VAR11;
VAR12; VAR13

Multiple R              0.94634     Maximum Log Likelihood              -68.14196
R-Square                0.89555     Akaike Info Criterion (AIC)           1.11920
Adjusted R-Square       0.88606     Bayes Schwarz Criterion (BSC)         1.38608
Standard Error          0.40566     Hannan-Quinn Criterion (HQC)          1.22764
Observations                145

                Coeff     Std. Error     T-stat     P-value     Lower 5%    Upper 95%
Intercept      0.06398     0.22003      0.29079     0.77166     -0.37126     0.49923
VAR1           0.44263     0.25668      1.72449     0.08696     -0.06510     0.95037
VAR2           0.09986     0.05363      1.86214     0.06481     -0.00622     0.20594
VAR3           0.14465     0.20251      0.71428     0.47632     -0.25594     0.54523
VAR5           0.06306     0.03902      1.61599     0.10848     -0.01413     0.14025
VAR6           0.22637     0.04861      4.65714     0.00001      0.13022     0.32252
VAR7          -0.27376     0.24800     -1.10386     0.27166     -0.76432     0.21681
VAR8          -0.10016     0.28012     -0.35758     0.72123     -0.65426     0.45393
VAR9           0.80285     0.05408     14.84537     0.00000      0.69587     0.90983
VAR10         -0.08186     0.04092     -2.00059     0.04749     -0.16280    -0.00092
VAR11          0.03753     0.05026      0.74679     0.45652     -0.06188     0.13695
VAR12          0.07312     0.18730      0.39041     0.69686     -0.29737     0.44362
VAR13         -0.17527     0.06433     -2.72478     0.00731     -0.30251    -0.04803

ANOVA
                    DF              SS             MS          F-Stat          p-Value
Regression          12          186.25          15.52        94.31674          0.00000
Residual           132           21.72           0.16
Total              144          207.97

Hypothesis Test
  Critical F-statistic (99% confidence with DFR1 and DFR2): 2.322190
  Critical F-statistic (95% confidence with DFR1 and DFR2): 1.826197
  Critical F-statistic (90% confidence with DFR1 and DFR2): 1.596134
```

Conclusion: A quick linear regression model on all the variables indicates that not all variables can statistically significantly explain the perceived ROI value to a supervisor sending employees to specific training courses. Additional analysis is required to determine the statistically critical variables.

Basic Econometrics and Regression

```
Model Inputs: VAR4 vs. LN(VAR2); VAR6; VAR9; VAR13

Multiple R             0.94580     Maximum Log Likelihood           -68.84037
R-Square               0.89454     Akaike Info Criterion (AIC)        1.01849
Adjusted R-Square      0.89152     Bayes Schwarz Criterion (BSC)      1.12113
Standard Error         0.39582     Hannan-Quinn Criterion (HQC)       1.06020
Observations               145

                  Coeff     Std. Error    T-stat      P-value    Lower 5%    Upper 95%
Intercept       -0.22577     0.19891    -1.13507     0.25829    -0.61902     0.16748
LN(VAR2)         0.76059     0.19318     3.93730     0.00013     0.37867     1.14251
VAR6             0.22195     0.04354     5.09750     0.00000     0.13587     0.30803
VAR9             0.83622     0.04443    18.82177     0.00000     0.74838     0.92406
VAR13           -0.22733     0.06006    -3.78502     0.00023    -0.34608    -0.10859



ANOVA
                      DF              SS            MS             F       p-Value
Regression             4          186.04         46.51     296.86391       0.00000
Residual             140           21.93          0.16
Total                144          207.97

Hypothesis Test
  Critical F-statistic (99% confidence with DFR1 and DFR2): 3.456075
  Critical F-statistic (95% confidence with DFR1 and DFR2): 2.436317
  Critical F-statistic (90% confidence with DFR1 and DFR2): 1.985450
```

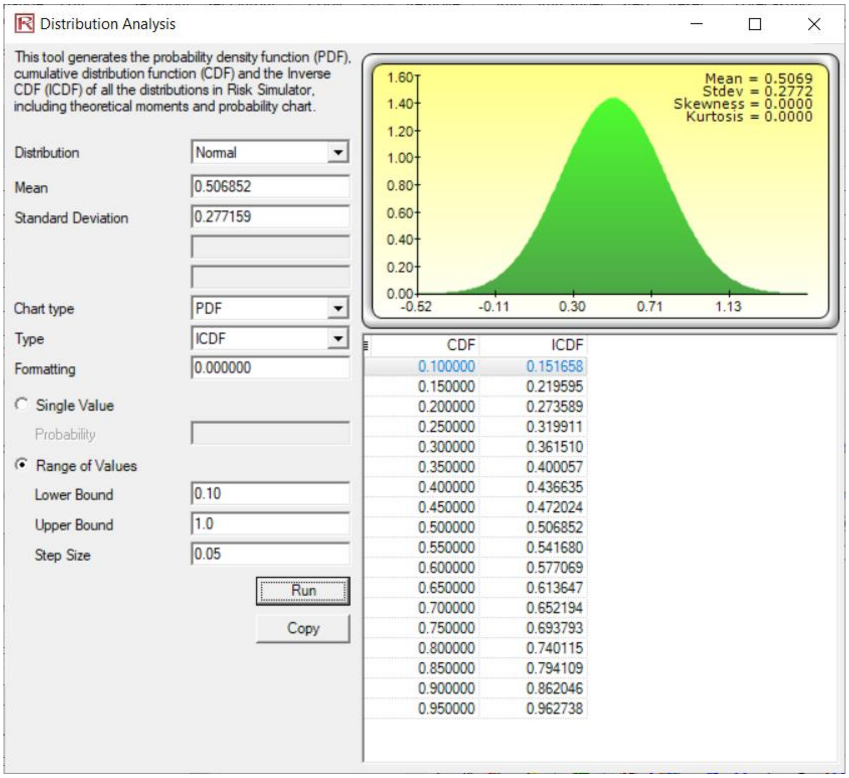
Conclusion: After employing an auto-econometric model, we conclude that the four variables above statistically significantly explain the perceived ROI of a training course to a supervisor. Specifically, the supervisor sees value if the training helped improved an employee's performance and was able to successfully apply the knowledge and skills, but only if the training is also worthwhile to the employee's own career development based on specific goals and expectations set prior to the training. Each of these variables by itself does not necessarily contribute to the perceived ROI compared to when they are combined holistically.

Distributional Fitting: Continuous (Anderson-Darling)

```
Rank      MAPE %          AD    Distribution
____    ________    ________    ___________
   1      13.47%      0.1976    Normal
   2      15.37%      0.2108    Logistic
   3      16.68%      0.3170    GumbelMax
   4      27.51%      0.2899    GumbelMin
   5     132.17%      0.6781    TDist
   6     223.22%      1.0758    Weibull3
   7     223.30%      1.0277    Standard Normal
   8     367.44%      0.3224    Exponential2
   9    1446.22%         N/A    Uniform
```

```
Best Fit Rank: 1
Fit Name: Normal
Anderson-Darling Statistic: 0.197647
MAPE: 0.134716
Mean: 0.506852
Sigma: 0.277159
Actual to Theoretical Four Moments:
  0.512414   0.264282   0.028672  -0.771227
  0.506852   0.277159   0.000000   0.000000

Best Fit Rank: 2
Fit Name: Logistic
Alpha: 0.503744
Anderson-Darling Statistic: 0.210849
Beta: 0.159402
MAPE: 0.153696
Actual to Theoretical Four Moments:
  0.512414   0.264282   0.028672  -0.771227
  0.503744   0.289124   0.000000   1.200000

Best Fit Rank: 3
Fit Name: GumbelMax
Alpha: 0.358716
Anderson-Darling Statistic: 0.317010
Beta: 0.273363
MAPE: 0.166753
Actual to Theoretical Four Moments:
  0.512414   0.264282   0.028672  -0.771227
  0.516506   0.350601   1.139547   2.400000

Best Fit Rank: 4
Fit Name: GumbelMin
Alpha: 0.642575
Anderson-Darling Statistic: 0.289929
Beta: 0.255307
MAPE: 0.275135
Actual to Theoretical Four Moments:
  0.512414   0.264282   0.028672  -0.771227
  0.495208   0.327444  -1.139547   2.400000

Best Fit Rank: 5
Fit Name: TDist
Anderson-Darling Statistic: 0.678070
Df: 26.000000
Location: 0.512414
MAPE: 1.321729
Precision: 0.000000
Actual to Theoretical Four Moments:
  0.512414   0.264282   0.028672  -0.771227
  0.512414   1.040833   0.000000   0.272727

Best Fit Rank: 6
Fit Name: Weibull3
Alpha: 2.000000
Anderson-Darling Statistic: 1.075775
Beta: 11.404554
Location: 10.000001
MAPE: 2.232214
Actual to Theoretical Four Moments:
  0.512414   0.264282   0.028672  -0.771227
 20.107024   5.283175   0.631111   0.245089
```

```
Best Fit Rank: 7
Fit Name: Standard Normal
Anderson-Darling Statistic: 1.027749
MAPE: 2.232990
Mean: 0.000000
Sigma: 1.000000
Actual to Theoretical Four Moments:
  0.512414   0.264282   0.028672  -0.771227
  0.000000   1.000000   0.000000   0.000000

Best Fit Rank: 8
Fit Name: Exponential2
Anderson-Darling Statistic: 0.322358
Lambda: 3.200114
Location: 0.248132
MAPE: 3.674428
Actual to Theoretical Four Moments:
  0.512414   0.264282   0.028672  -0.771227
  0.560621   0.312489   2.000000   6.000000

Best Fit Rank: 9
Fit Name: Uniform
Anderson-Darling Statistic: N/A
MAPE: 14.462180
Max: 3.618024
Min: 2.618024
Actual to Theoretical Four Moments:
  0.512414   0.264282   0.028672  -0.771227
  3.118024   0.288675   0.000000  -1.200000

Correlation Matrix :
  1.000000

Data Fitting and Simulation:
  0.536962
```



This tool generates the probability density function (PDF), cumulative distribution function (CDF) and the Inverse CDF (ICDF) of all the distributions in Risk Simulator, including theoretical moments and probability chart.

Distribution: Normal
Mean: 0.506852
Standard Deviation: 0.277159
Chart type: PDF
Type: ICDF
Formatting: 0.000000
○ Single Value
Probability:
● Range of Values
Lower Bound: 0.10
Upper Bound: 1.0
Step Size: 0.05

CDF	ICDF
0.100000	0.151658
0.150000	0.219595
0.200000	0.273589
0.250000	0.319911
0.300000	0.361510
0.350000	0.400057
0.400000	0.436635
0.450000	0.472024
0.500000	0.506852
0.550000	0.541680
0.600000	0.577069
0.650000	0.613647
0.700000	0.652194
0.750000	0.693793
0.800000	0.740115
0.850000	0.794109
0.900000	0.862046
0.950000	0.962738

STUDENTS’ POINT OF VIEW

- VAR1. Follow-up Survey: How critical was applying the content of the training to your job success?
- VAR2. Post-event Survey: How critical is applying the content of this training to your job success?
- VAR3. Follow-up Survey: What percent of new knowledge and skills learned from this training did you directly apply to your job?
- VAR4. Post-event Survey: What percent of new knowledge and skills learned from this training do you estimate you will directly apply to your job?
- VAR5. Follow-up Survey: What percent of your total work time have you spent on tasks that require the knowledge/skills presented in the training?
- VAR6. Post-event Survey: What percent of your total work time requires the knowledge or skills presented in this training?
- VAR7. Follow-up Survey: Estimate how much of the improvement was a direct result of this training.
- VAR8. Follow-up Survey: I have been able to successfully apply the knowledge/skills learned in this class to my job.
- VAR9. Follow-up Survey: I have learned new knowledge/skills from this training.

Inter Class Correlation for Inter-rater Reliability Test

```
Model Inputs: VAR1; VAR2; VAR3; VAR4; VAR5; VAR6; VAR7

              DF        Sums of Squares    Mean Square        F-Stat         p-Value
Rows          16141     4144.78            0.26               4.46920        0.00000
Columns       6         119.88             19.98              347.72605      0.00000
Error         96846     5564.45            0.06
Total         112993    9829.11

Interclass Correlation   0.32668
```

A high ICC indicates a high level of reliability vs. low correlations mean low reliability and low consistency.

The null hypothesis indicates that there is zero statistical reliability among the seven survey questions requiring percentage responses.

Conclusion: There is statistical reliability among the percentage responses. This means that for the 16,157 students who responded to the survey, the responses as a whole exhibited statistical reliability. We can conclude that the responses to the survey are valid and trustworthy, rather than being completed haphazardly.

Inter Class Correlation for Inter-rater Reliability Test

```
Model Inputs: VAR1; VAR3; VAR5; VAR7

              DF        Sums of Squares    Mean Square        F-Stat         p-Value
Rows          16141     4501.85            0.28               12.95516       0.00000
Columns       3         38.93              12.98              602.77198      0.00000
Error         48423     1042.48            0.02
Total         64567     5583.26

Interclass Correlation   0.74236
```

A high ICC indicates a high level of reliability vs. low correlations mean low reliability and low consistency.

The null hypothesis indicates that there is zero statistical reliability among the four follow-up survey questions requiring percentage responses.

Conclusion: There is statistical reliability among the percentage responses. This means that for the 16,157 students who responded to the follow-up survey, the responses as a whole exhibited statistical reliability. We can conclude that the responses to the follow-up survey are valid and trustworthy, rather than being completed haphazardly.

Inter Class Correlation for Inter-rater Reliability Test

```
Model Inputs: VAR2; VAR4; VAR6

              DF        Sums of Squares    Mean Square          F-Stat        p-Value
Rows          16141     3729.17            0.23                 17.11696      0.00000
Columns       2         23.24              11.62                860.85623     0.00000
Error         32282     435.73             0.01
Total         48425     4188.14

Interclass Correlation  0.83608
```

A high ICC indicates a high level of reliability vs. low correlations mean low reliability and low consistency.

The null hypothesis indicates that there is zero statistical reliability among the three survey questions requiring percentage responses that were asked immediately after the conclusion of the course.

Conclusion: There is statistical reliability among the percentage responses. This means that for the 16,157 students who responded to the end-of-course survey, the responses as a whole exhibited statistical reliability. We can conclude that the responses to the follow-up survey are valid and trustworthy, rather than being completed haphazardly.

Inter Class Correlation for Inter-rater Reliability Test

```
Model Inputs: VAR8; VAR9

              DF        Sums of Squares    Mean Square          F-Stat        p-Value
Rows          16141     32933.22           2.04                 1.04379       0.00324
Columns       1         249.48             249.48               127.62848     0.00000
Error         16141     31551.52           1.95
Total         32283     64734.22

Interclass Correlation  0.02126
```

A low ICC indicates a low level of reliability and low correlations mean low reliability and low consistency.

The null hypothesis indicates that there is zero statistical reliability between the follow-up responses of the quantity of new knowledge learned versus knowledge actually applied.

Conclusion: There is no statistical reliability among the percentage responses. This means that for the 16,157 students who responded to the follow-up survey, the responses as a whole exhibited no reliability with respect to the amount of knowledge learned and retained versus the amount of knowledge actually applied in their jobs.

Guttman’s Lambda & Internal Consistency and Reliability Test

```
Model Inputs: VAR1; VAR3; VAR5; VAR7

Covariance                          0.25622
Variance of Total                   1.11563
Guttman’s Lambda                    0.91866

Split Half Approach

Correlation Coefficient             0.86983
Spearman-Brown Correction           0.93038

Odd-Even Split Approach

Correlation Coefficient             0.84957
Spearman-Brown Correction           0.91867
```

High correlations and lambda scores mean high reliability and high consistency.

The null hypothesis states that there is zero statistical consistency among the four follow-up survey questions requiring percentage responses.

Conclusion: There is statistical consistency among the follow-up survey questions. We can conclude that the survey data are statistically valid and can be trusted.

Guttman’s Lambda & Internal Consistency and Reliability Test

```
Model Inputs: VAR2; VAR4; VAR6

Covariance                          0.14515
Variance of Total                   0.69311
Guttman’s Lambda                    0.83769

Split Half Approach

Correlation Coefficient             0.92573
Spearman-Brown Correction           0.96143


Odd-Even Split Approach

Correlation Coefficient             0.87888
Spearman-Brown Correction           0.93553
```

High correlations and lambda scores mean high reliability and high consistency.

The null hypothesis states that there is zero statistical consistency among the three end-of-course survey questions requiring percentage responses.

Conclusion: There is statistical consistency among the end-of-course survey questions. We can conclude that the survey data are statistically valid and can be trusted.

Guttman’s Lambda & Internal Consistency and Reliability Test

```
Model Inputs: VAR8; VAR9

Covariance                     0.04280
Variance of Total              4.08069
Guttman’s Lambda               0.04195

Split Half Approach

Correlation Coefficient        0.02144
Spearman-Brown Correction      0.04198

Odd-Even Split Approach

Correlation Coefficient        0.02144
Spearman-Brown Correction      0.04198

Low correlations and lambda scores mean low reliability and low consistency.
```

The null hypothesis indicates that there is zero statistical reliability between the follow-up responses of the quantity of new knowledge learned versus knowledge actually applied.

Conclusion: There is no statistical reliability among the percentage responses. This means that for the 16,157 students who responded to the follow-up survey, the responses as a whole exhibited no reliability with respect to the amount of knowledge learned and retained versus the amount of knowledge actually applied in their jobs.

One Way ANOVA with Randomized Multiple Treatments

```
Model Inputs: VAR1; VAR2; VAR3; VAR4; VAR5; VAR6; VAR7

                      DF          Sums of Sq.    Mean Square      F Stat     p-Value
Between Groups         6             119.88         19.98       232.4993     0.0000
Within Groups     112987            9709.23          0.09
Total             112993            9829.11          0.09

F Critical @ 0.10               1.774159
F Critical @ 0.05               2.098678
F Critical @ 0.01               2.802141
```

The null hypothesis tested is such that all the survey questions requiring percentage inputs are statistically equivalent.

Conclusion: The hypothesis is rejected, indicating that at least one of the variables is different from the rest. To identify the differences, additional analysis is required.

Kruskal-Wallis Test

```
H Statistic: 1225.824397
p-Value: 0.000000
H Critical at 1%: 16.811894
H Critical at 5%: 12.591587
H Critical at 10%: 10.644641
The population medians are statistically not equal at 1%, 5%, or 10% significance
```

Two Variable (T) Independent Equal Variance

```
Model Inputs: VAR1; VAR2

Column 1 Observations: 16142
Column 1 Sample Mean: 0.475449
Column 1 Sample Standard Deviation: 0.301538
Column 2 Observations: 16142
Column 2 Sample Mean: 0.507124
Column 2 Sample Standard Deviation: 0.301758
Sample Mean Difference: -0.031675
t-Statistic: -9.433703
Hypothesized Mean: 0.000000
p-Value Two Tailed: 0.000000
```

The null hypothesis tested is such that the criticality of the materials learned in class as perceived immediately after the class is equal to the updated perception in the future when a follow-up survey is conducted.

Conclusion: This is found to be statistically significantly different, indicating that the student's view of the usefulness of the course material presented is materially and significantly different after spending time on the job.

Two Variable (T) Independent Equal Variance

```
Model Inputs: VAR3; VAR4
Column 1 Observations: 16142
Column 1 Sample Mean: 0.438508
Column 1 Sample Standard Deviation: 0.298174
Column 2 Observations: 16142
Column 2 Sample Mean: 0.494146
Column 2 Sample Standard Deviation: 0.293491
Sample Mean Difference: -0.055637
t-Statistic: -16.895509
Hypothesized Mean: 0.000000
p-Value Two Tailed: 0.000000
```

The null hypothesis tested is such that the amount of new knowledge learned that might be applicable to their job as perceived immediately after the class is equal to the updated perception in the future when a follow-up survey is conducted.

Conclusion: This is found to be statistically significantly different, indicating that the student's view of the amount of new knowledge learned that might be applicable to their job is materially and significantly different after spending time on the job.

Two Variable (T) Independent Equal Variance

```
Model Inputs: VAR5; VAR6
Column 1 Observations: 16142
Column 1 Sample Mean: 0.406046
Column 1 Sample Standard Deviation: 0.280724
Column 2 Observations: 16142
Column 2 Sample Mean: 0.455545
Column 2 Sample Standard Deviation: 0.284319
Sample Mean Difference: -0.049498
t-Statistic: -15.739547
Hypothesized Mean: 0.000000
p-Value Two Tailed: 0.000000
```

The null hypothesis tested is such that the amount of work time requiring the use of the new knowledge learned as perceived immediately after the class is equal to the updated perception in the future when a follow-up survey is conducted.

Conclusion: This is found to be statistically significantly different, indicating that the student’s view of the amount of work time requiring the use of the new knowledge learned is materially and significantly different after spending time on the job.

```
--------------------------------------------------------------------------------------

Nonparametric Mann-Whitney Test
Two Independent Samples
Model Inputs: VAR1; VAR2

                    Sample 1            Sample 2
Count               16142               16142
Median              0.50                0.50
Rank Sum            252813347.00        268331123.00
U Values            138040970.00        122523194.00

Wilcoxon W          252813347.00
U-Stat              122523194.00
Mean                130282082.00
Std Dev             837273.05556
Z-Score             9.26685
P-value (One Tail)  0.00000
P-value (Two Tail)  0.00000             * Adjusted for Ties

Null hypothesis: There is zero difference between the two variables.

--------------------------------------------------------------------------------------

Nonparametric Mann-Whitney Test
Two Independent Samples
Model Inputs: VAR3; VAR4

                    Sample 1            Sample 2
Count               16142               16142
Median              0.40                0.50
Rank Sum            246343176.50        274801293.50
U Values            144511140.50        116053023.50

Wilcoxon W          246343176.50
U-Stat              116053023.50
Mean                130282082.00
Std Dev             837273.05556
Z-Score             16.99453
P-value (One Tail)  0.00000
P-value (Two Tail)  0.00000             * Adjusted for Ties

Null hypothesis: There is zero difference between the two variables.

--------------------------------------------------------------------------------------

Nonparametric Mann-Whitney Test
Two Independent Samples
Model Inputs: VAR5; VAR6

                    Sample 1            Sample 2
Count               16142               16142
Median              0.40                0.50
Rank Sum            247430876.00        273713594.00
U Values            143423441.00        117140723.00
```

```
Wilcoxon W          247430876.00
U-Stat              117140723.00
Mean                130282082.00
Std Dev             837273.05556
Z-Score             15.69543
P-value (One Tail)  0.00000
P-value (Two Tail)  0.00000             * Adjusted for Ties

Null hypothesis: There is zero difference between the two variables.
--------------------------------------------------------------------------------------
```

One Variable (T) Mean

```
Model Inputs: VAR7
Observations: 16142
Hypothesized Mean: 0.000000
Sample Mean: 0.439753
Standard Deviation (Sample): 0.291298
t-Statistic: 191.800566
p-Value Right Tailed: 0.000000
```

The null hypothesis tested is such that any improvement at work was a not a result of the training.

Conclusion: There is statistically significant improvement in the student's work abilities as a direct result of the training received.

```
--------------------------------------------------------------------------------------
```

One Variable (T) Mean

```
Model Inputs: VAR8
Observations: 16142
Hypothesized Mean: 3.500000
Sample Mean: 5.505018
Standard Deviation (Sample): 1.436794
t-Statistic: 177.297469
p-Value Right Tailed: 0.000000
```

The null hypothesis tested is such that there is zero ability to apply the knowledge and skills learned in class.

Conclusion: There is statistically significant ability to apply the knowledge and skills learned in class.

```
--------------------------------------------------------------------------------------
```

One Variable (T) Mean

```
Model Inputs: VAR9
Observations: 16142
Hypothesized Mean: 3.500000
Sample Mean: 5.680833
Standard Deviation (Sample): 1.389501
t-Statistic: 199.407765
p-Value Right Tailed: 0.000000
```

The null hypothesis tested is such that there is zero new knowledge learned in class.

Conclusion: There is a statistically significant amount of new knowledge learned in class.

```
--------------------------------------------------------------------------------------
```

Stepwise Regression (Forward-Backward)

```
Model Inputs: VAR7 vs. VAR1; VAR2; VAR3; VAR4; VAR5; VAR6; VAR8; VAR9

Forward Method: Errors of Y and Highest correlation of Xs

Highest absolute correlation value: 0.724073

ARRANGEMENT: Y<->X3

Regression Results

OVERALL FIT
Multiple R               0.72407     Maximum Log Likelihood              3001.84659
R-Square                 0.52428     Akaike Info Criterion (AIC)           -0.37168
Adjusted R-Square        0.52425     Bayes Schwarz Criterion (BSC)         -0.37073
Standard Error           0.20092     Hannan-Quinn Criterion (HQC)          -0.37137
Observations               16142

                   Coeff  Std. Error      T-stat     P-value    Lower 5%   Upper 95%
Intercept        0.12956     0.00281    46.06698     0.00000     0.12405     0.13508
X3               0.70738     0.00530   133.37046     0.00000     0.69698     0.71777

ANOVA
                      DF          SS          MS           F     p-Value
Regression             1      718.08      718.08 17787.67983     0.00000
Residual           16140      651.56        0.04
Total              16141     1369.64

Hypothesis Test
  Critical F-statistic (99% confidence with DFR1 and DFR2): 6.636466
  Critical F-statistic (95% confidence with DFR1 and DFR2): 3.842035
  Critical F-statistic (90% confidence with DFR1 and DFR2): 2.705854

Highest absolute correlation value: 0.161817

ARRANGEMENT: Y<->X3;X7

Regression Results

OVERALL FIT
Multiple R               0.73691     Maximum Log Likelihood              3326.39860
R-Square                 0.54303     Akaike Info Criterion (AIC)           -0.41177
Adjusted R-Square        0.54298     Bayes Schwarz Criterion (BSC)         -0.41034
Standard Error           0.19693     Hannan-Quinn Criterion (HQC)          -0.41130
Observations               16142

                   Coeff  Std. Error      T-stat     P-value    Lower 5%   Upper 95%
Intercept       -0.01624     0.00630    -2.57785     0.00995    -0.02859    -0.00389
X3               0.61229     0.00638    96.00230     0.00000     0.59979     0.62479
X7               0.03406     0.00132    25.73341     0.00000     0.03147     0.03665

ANOVA
                      DF          SS          MS           F     p-Value
Regression             2      743.76      371.88  9589.29877     0.00000
Residual           16139      625.88        0.04
Total              16141     1369.64

Hypothesis Test
  Critical F-statistic (99% confidence with DFR1 and DFR2): 4.606484
  Critical F-statistic (95% confidence with DFR1 and DFR2): 2.996288
  Critical F-statistic (90% confidence with DFR1 and DFR2): 2.302914
```

```
Highest absolute correlation value: 0.119257

ARRANGEMENT: Y<->X3;X7;X1

Regression Results

OVERALL FIT
Multiple R             0.75217      Maximum Log Likelihood             3738.14031
R-Square               0.56576      Akaike Info Criterion (AIC)          -0.46266
Adjusted R-Square      0.56568      Bayes Schwarz Criterion (BSC)        -0.46076
Standard Error         0.19197      Hannan-Quinn Criterion (HQC)         -0.46203
Observations             16142


                   Coeff  Std. Error      T-stat     P-value    Lower 5%   Upper 95%
Intercept       -0.01137     0.00614    -1.84963     0.06439    -0.02341     0.00068
X3               0.40320     0.00951    42.40337     0.00000     0.38456     0.42184
X7               0.02631     0.00132    19.96690     0.00000     0.02373     0.02889
X1               0.27235     0.00937    29.06271     0.00000     0.25398     0.29071

ANOVA
                      DF          SS          MS           F     p-Value
Regression             3      774.89      258.30  7008.58981     0.00000
Residual           16138      594.75        0.04
Total              16141     1369.64

Hypothesis Test
  Critical F-statistic (99% confidence with DFR1 and DFR2): 3.782835
  Critical F-statistic (95% confidence with DFR1 and DFR2): 2.605459
  Critical F-statistic (90% confidence with DFR1 and DFR2): 2.084135

Highest absolute correlation value: 0.024098

ARRANGEMENT: Y<->X3;X7;X1;X5

Regression Results

OVERALL FIT
Multiple R             0.75271      Maximum Log Likelihood             3753.34608
R-Square               0.56658      Akaike Info Criterion (AIC)          -0.46442
Adjusted R-Square      0.56647      Bayes Schwarz Criterion (BSC)        -0.46204
Standard Error         0.19180      Hannan-Quinn Criterion (HQC)         -0.46363
Observations             16142


                   Coeff  Std. Error      T-stat     P-value    Lower 5%   Upper 95%
Intercept       -0.01274     0.00614    -2.07418     0.03808    -0.02479    -0.00070
X3               0.37981     0.01040    36.50831     0.00000     0.35942     0.40020
X7               0.02617     0.00132    19.87973     0.00000     0.02359     0.02875
X1               0.25276     0.01001    25.24347     0.00000     0.23314     0.27239
X5               0.05341     0.00968     5.51641     0.00000     0.03443     0.07239

ANOVA
                      DF          SS          MS           F     p-Value
Regression             4      776.01      194.00  5273.63619     0.00000
Residual           16137      593.63        0.04
Total              16141     1369.64

Hypothesis Test
  Critical F-statistic (99% confidence with DFR1 and DFR2): 3.320336
  Critical F-statistic (95% confidence with DFR1 and DFR2): 2.372483
  Critical F-statistic (90% confidence with DFR1 and DFR2): 1.945208
```

```
Highest absolute correlation value: 0.010031

ARRANGEMENT: Y<->X3;X7;X1;X5;X8

Regression Results

OVERALL FIT
Multiple R             0.75274      Maximum Log Likelihood             3754.15899
R-Square               0.56662      Akaike Info Criterion (AIC)          -0.46440
Adjusted R-Square      0.56649      Bayes Schwarz Criterion (BSC)        -0.46154
Standard Error         0.19180      Hannan-Quinn Criterion (HQC)         -0.46345
Observations             16142

                   Coeff   Std. Error     T-stat     P-value    Lower 5%   Upper 95%
Intercept       -0.02045     0.00862    -2.37282     0.01766    -0.03734    -0.00356
X3               0.38002     0.01040    36.52469     0.00000     0.35963     0.40042
X7               0.02614     0.00132    19.85173     0.00000     0.02356     0.02872
X1               0.25249     0.01002    25.21088     0.00000     0.23286     0.27212
X5               0.05352     0.00968     5.52807     0.00000     0.03454     0.07250
X8               0.00139     0.00109     1.27487     0.20237    -0.00074     0.00352

ANOVA
                      DF          SS          MS           F     p-Value
Regression             5      776.07      155.21  4219.39750     0.00000
Residual           16136      593.57        0.04
Total              16141     1369.64

Hypothesis Test
  Critical F-statistic (99% confidence with DFR1 and DFR2): 3.018385
  Critical F-statistic (95% confidence with DFR1 and DFR2): 2.214653
  Critical F-statistic (90% confidence with DFR1 and DFR2): 1.847628
```

Backward Method

```
ARRANGEMENT: Y<->X3;X7;X1;X5;X8

Regression Results

OVERALL FIT
Multiple R             0.75274      Maximum Log Likelihood             3754.15899
R-Square               0.56662      Akaike Info Criterion (AIC)          -0.46403
Adjusted R-Square      0.56649      Bayes Schwarz Criterion (BSC)        -0.45974
Standard Error         0.19180      Hannan-Quinn Criterion (HQC)         -0.46261
Observations             16142

                   Coeff  Std. Error      T-stat     P-value    Lower 5%   Upper 95%
Intercept       -0.02045     0.00862    -2.37282     0.01766    -0.03734    -0.00356
X3               0.38002     0.01040    36.52469     0.00000     0.35963     0.40042
X7               0.02614     0.00132    19.85173     0.00000     0.02356     0.02872
X1               0.25249     0.01002    25.21088     0.00000     0.23286     0.27212
X5               0.05352     0.00968     5.52807     0.00000     0.03454     0.07250
X8               0.00139     0.00109     1.27487     0.20237    -0.00074     0.00352

ANOVA
                      DF          SS          MS           F     p-Value
Regression             5      776.07      155.21  4219.39750     0.00000
Residual           16136      593.57        0.04
Total              16141     1369.64

Hypothesis Test
  Critical F-statistic (99% confidence with DFR1 and DFR2): 3.018385
  Critical F-statistic (95% confidence with DFR1 and DFR2): 2.214653
  Critical F-statistic (90% confidence with DFR1 and DFR2): 1.847628
```

Backward Method

```
ARRANGEMENT: Y<->X3;X7;X1;X5
```

Regression Results

```
OVERALL FIT
Multiple R              0.75271     Maximum Log Likelihood            3753.34608
R-Square                0.56658     Akaike Info Criterion (AIC)         -0.46393
Adjusted R-Square       0.56647     Bayes Schwarz Criterion (BSC)       -0.45964
Standard Error          0.19180     Hannan-Quinn Criterion (HQC)        -0.46251
Observations              16142


                   Coeff  Std. Error      T-stat     P-value    Lower 5%   Upper 95%
Intercept       -0.01274     0.00614    -2.07418     0.03808    -0.02479    -0.00070
VAR3             0.37981     0.01040    36.50831     0.00000     0.35942     0.40020
VAR8             0.02617     0.00132    19.87973     0.00000     0.02359     0.02875
VAR1             0.25276     0.01001    25.24347     0.00000     0.23314     0.27239
VAR5             0.05341     0.00968     5.51641     0.00000     0.03443     0.07239


ANOVA
                      DF          SS          MS           F     p-Value
Regression             4      776.01      194.00  5273.63619     0.00000
Residual           16137      593.63        0.04
Total              16141     1369.64

Hypothesis Test
  Critical F-statistic (99% confidence with DFR1 and DFR2): 3.320336
  Critical F-statistic (95% confidence with DFR1 and DFR2): 2.372483
  Critical F-statistic (90% confidence with DFR1 and DFR2): 1.945208
```

Conclusion: At a future follow-up session, a former student's estimate of how much work improvement was a direct result of the training course depended on actual experience during the follow-up session and not immediately after the course ended.

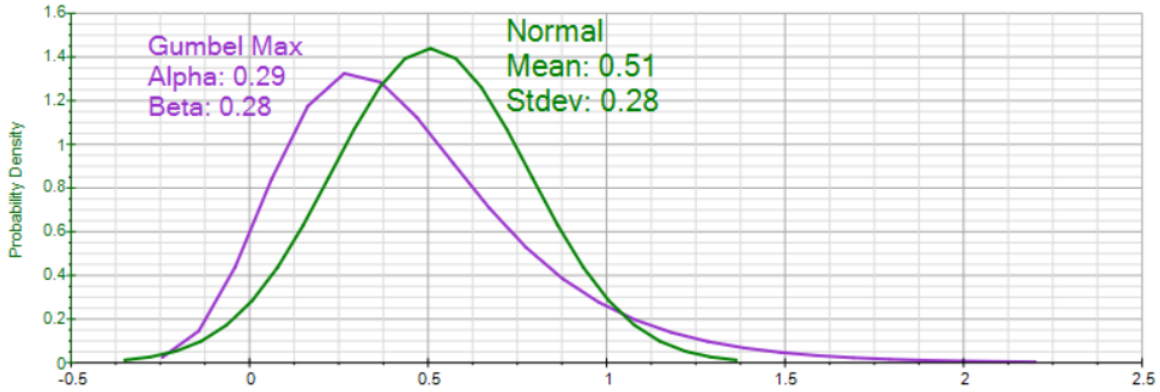
```
-------------------------------------------------------------------------------------
```

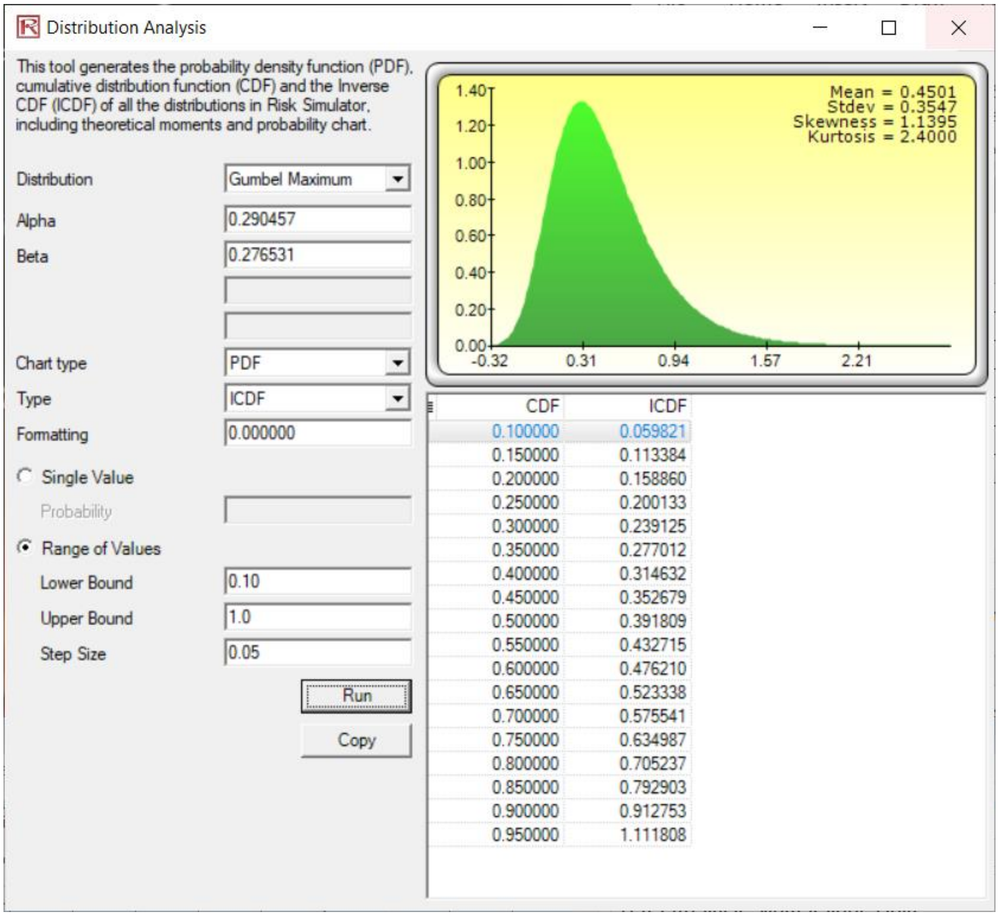
Distributional Fitting: Continuous (Anderson-Darling)

```
Rank      MAPE %          AD    Distribution
____    ________    ________    ___________
   1      45.80%      0.2826    GumbelMax
   2      46.98%      0.4680    Fréchet
   3      53.94%      0.2703    Normal
   4      57.65%      0.2782    Logistic
   5      88.72%      0.3492    GumbelMin
   6     289.64%      0.7048    TDist
   7     447.11%      1.0000    Standard Normal
   8     477.33%      1.0758    Weibull3
   9     551.54%      0.4355    Exponential2
  10    2710.10%         N/A    Uniform

Best Fit Rank: 1
Fit Name: GumbelMax
Alpha: 0.290457
Anderson-Darling Statistic: 0.282634
Beta: 0.276531
MAPE: 0.458042
```

```
Actual to Theoretical Four Moments:
  0.439753   0.291298   0.234879  -0.931816
  0.450074   0.354664   1.139547   2.400000

Best Fit Rank: 2
Fit Name: Fréchet
Alpha: 1.024787
Anderson-Darling Statistic: 0.467962
Beta: 0.218338
Location: -0.060247
MAPE: 0.469841
Actual to Theoretical Four Moments:
  0.439753   0.291298   0.234879  -0.931816


Best Fit Rank: 3
Fit Name: Normal
Anderson-Darling Statistic: 0.270312
MAPE: 0.539385
Mean: 0.434077
Sigma: 0.291298
Actual to Theoretical Four Moments:
  0.439753   0.291298   0.234879  -0.931816
  0.434077   0.291298   0.000000   0.000000

Best Fit Rank: 4
Fit Name: Logistic
Alpha: 0.425708
Anderson-Darling Statistic: 0.278228
Beta: 0.166381
MAPE: 0.576508
Actual to Theoretical Four Moments:
  0.439753   0.291298   0.234879  -0.931816
  0.425708   0.301782   0.000000   1.200000

Best Fit Rank: 5
Fit Name: GumbelMin
Alpha: 0.604820
Anderson-Darling Statistic: 0.349248
Beta: 0.279744
MAPE: 0.887160
Actual to Theoretical Four Moments:
  0.439753   0.291298   0.234879  -0.931816
  0.443347   0.358786  -1.139547   2.400000

Best Fit Rank: 6
Fit Name: TDist
Anderson-Darling Statistic: 0.704787
Df: 26.000000
Location: 0.439753
MAPE: 2.896411
Precision: 0.000000
Actual to Theoretical Four Moments:
  0.439753   0.291298   0.234879  -0.931816
  0.439753   1.040833   0.000000   0.272727

Best Fit Rank: 7
Fit Name: Standard Normal
Anderson-Darling Statistic: 1.000000
MAPE: 4.471108
Mean: 0.000000
Sigma: 1.000000
```

```
Actual to Theoretical Four Moments:
  0.439753   0.291298   0.234879  -0.931816
  0.000000   1.000000   0.000000   0.000000

Best Fit Rank: 8
Fit Name: Weibull3
Alpha: 2.000000
Anderson-Darling Statistic: 1.075775
Beta: 11.404554
Location: 10.000001
MAPE: 4.773270
Actual to Theoretical Four Moments:
  0.439753   0.291298   0.234879  -0.931816
 20.107024   5.283175   0.631111   0.245089

Best Fit Rank: 9
Fit Name: Exponential2
Anderson-Darling Statistic: 0.435484
Lambda: 3.334983
Location: 0.148455
MAPE: 5.515398
Actual to Theoretical Four Moments:
  0.439753   0.291298   0.234879  -0.931816
  0.448307   0.299852   2.000000   6.000000

Best Fit Rank: 10
Fit Name: Uniform
Anderson-Darling Statistic: N/A
MAPE: 27.100960
Max: 3.618024
Min: 2.618024
Actual to Theoretical Four Moments:
  0.439753   0.291298   0.234879  -0.931816
  3.118024   0.288675   0.000000  -1.200000

Correlation Matrix :
  1.000000

Data Fitting and Simulation:
  0.427064
```





This tool generates the probability density function (PDF), cumulative distribution function (CDF) and the Inverse CDF (ICDF) of all the distributions in Risk Simulator, including theoretical moments and probability chart.

Mean = 0.4501
Stdev = 0.3547
Skewness = 1.1395
Kurtosis = 2.4000

Distribution: Gumbel Maximum
Alpha: 0.290457
Beta: 0.276531
Chart type: PDF
Type: ICDF
Formatting: 0.000000
○ Single Value
Probability:
● Range of Values
Lower Bound: 0.10
Upper Bound: 1.0
Step Size: 0.05

CDF	ICDF
0.100000	0.059821
0.150000	0.113384
0.200000	0.158860
0.250000	0.200133
0.300000	0.239125
0.350000	0.277012
0.400000	0.314632
0.450000	0.352679
0.500000	0.391809
0.550000	0.432715
0.600000	0.476210
0.650000	0.523338
0.700000	0.575541
0.750000	0.634987
0.800000	0.705237
0.850000	0.792903
0.900000	0.912753
0.950000	1.111808

DATA MINING & SUPERVISED LEARNING AI MODEL RESULTS

Random Forest Supervised Data Mining

```
Bagging with 100 iterations and base learner with Cross-validation

Correlation coefficient                  0.8659
Mean absolute error                      0.0923
Root mean squared error                  0.1470
Relative absolute error                 37.356%
Root relative squared error             50.091%
Total Number of Instances                16,142
```

---

k-Means Clustering

```
Number of iterations: 19
Within cluster sum of squared errors: 8084.545176982922

Initial starting points (random):

Cluster 0: 1,0.8,1,0.7,0.5,0.6,0.5,7,7
Cluster 1: 0.1,0.1,0.1,0.1,0.1,0.1,0.1,5,6

Missing values globally replaced with mean/mode

Final cluster centroids:
                               Cluster#
Attribute     Full Data        0          1
              (16142.0)    (8044.0)   (8098.0)
============================================
VAR1             0.4754     0.7143     0.2382
VAR2             0.5071     0.5019     0.5123
VAR3             0.4385     0.6783     0.2003
VAR4             0.4941     0.4882     0.5000
VAR5             0.4060     0.6141     0.1994
VAR6             0.4555     0.4495     0.4616
VAR7             0.4398     0.6393     0.2416
VAR8             5.5050     6.2471     4.7678
VAR9             5.6808     5.6875     5.6742
```

Multi-Layered Perceptron

```
Classifier model (full training set)

Linear Node 0
    Inputs          Weights
    Threshold       0.06925846705171
    Node 1          -0.9353491867299
    Node 2          1.00459405724956
    Node 3          1.58048358855907
    Node 4          -0.8778430933414
```

```
Sigmoid Node 1
    Inputs          Weights
    Threshold        -4.4047045440550
    Attrib VAR1      0.19969975101756
    Attrib VAR2      -3.9268483939591
    Attrib VAR3      1.16905258543428
    Attrib VAR5      -0.0219115990958
    Attrib VAR6      3.71003744856782
    Attrib VAR7      -0.8445600857485
    Attrib VAR8      -0.4089364871545
    Attrib VAR9      -0.3121450362066
Sigmoid Node 2
    Inputs          Weights
    Threshold       -2.28419832728941
    Attrib VAR1      0.05526161653994
    Attrib VAR2      2.03265857744563
    Attrib VAR3      0.08508072244526
    Attrib VAR5     -0.11467093508898
    Attrib VAR6      1.84118709331492
    Attrib VAR7     -0.14182140148667
    Attrib VAR8     -0.33365187693914
    Attrib VAR9      0.91602666389471
Sigmoid Node 3
    Inputs          Weights
    Threshold       -4.56333319155951
    Attrib VAR1     -0.25236427116584
    Attrib VAR2      0.74258700859387
    Attrib VAR3     -0.56585551066520
    Attrib VAR5      0.47073993316553
    Attrib VAR6      1.04094374767149
    Attrib VAR7      0.08495076816424
    Attrib VAR8      0.26774091586211
    Attrib VAR9      0.30093703703610
Sigmoid Node 4
    Inputs          Weights
    Threshold       -2.30364495804448
    Attrib VAR1     -0.12591050519854
    Attrib VAR2     -1.86519774207535
    Attrib VAR3     -0.18538069712247
    Attrib VAR5      0.09881940907619
    Attrib VAR6     -1.74070057924794
    Attrib VAR7      0.05141128188744
    Attrib VAR8      0.06166162977156
    Attrib VAR9     -0.84230040259725

Cross-validation
Correlation coefficient                    0.8260
Mean absolute error                        0.1167
Root mean squared error                    0.1702
Relative absolute error                   47.265%
Root relative squared error               57.999%
Total Number of Instances                  16,142
```

Random Tree Classification

```
Instances:     16142
Attributes:    9
Test mode:     10-fold cross-validation
```

```
VAR9 < 5.5
|   VAR6 < 0.35
|   |   VAR6 < 0.15
|   |   |   VAR2 < 0.05
|   |   |   |   VAR6 < 0.05
|   |   |   |   |   VAR9 < 2.5
|   |   |   |   |   |   VAR5 < 0.85
|   |   |   |   |   |   |   VAR7 < 0.25
|   |   |   |   |   |   |   |   VAR3 < 0.15
|   |   |   |   |   |   |   |   |   VAR1 < 0.05
|   |   |   |   |   |   |   |   |   |   VAR8 < 3.5: 0 (14/0)
|   |   |   |   |   |   |   |   |   |   VAR8 >= 3.5
|   |   |   |   |   |   |   |   |   |   |   VAR9 < 1.5: 0 (4/0)
|   |   |   |   |   |   |   |   |   |   |   VAR9 >= 1.5
|   |   |   |   |   |   |   |   |   |   |   |   VAR8 < 4.5: 0.05 (2/0)
|   |   |   |   |   |   |   |   |   |   |   |   VAR8 >= 4.5: 0 (1/0)
|   |   |   |   |   |   |   |   |   VAR1 >= 0.05: 0 (23/0)
|   |   |   |   |   |   |   |   VAR3 >= 0.15
|   |   |   |   |   |   |   |   |   VAR8 < 5.5
|   |   |   |   |   |   |   |   |   |   VAR5 < 0.25
|   |   |   |   |   |   |   |   |   |   |   VAR7 < 0.05: 0 (3/0)
|   |   |   |   |   |   |   |   |   |   |   VAR7 >= 0.05
|   |   |   |   |   |   |   |   |   |   |   |   VAR9 < 1.5: 0 (6/0)
|   |   |   |   |   |   |   |   |   |   |   |   VAR9 >= 1.5
|   |   |   |   |   |   |   |   |   |   |   |   |   VAR5 < 0.15: 0.1 (3/-0)
|   |   |   |   |   |   |   |   |   |   |   |   |   VAR5 >= 0.15: 0 (2/0)
|   |   |   |   |   |   |   |   |   |   VAR5 >= 0.25
|   |   |   |   |   |   |   |   |   |   |   VAR1 < 0.25
|   |   |   |   |   |   |   |   |   |   |   |   VAR7 < 0.15: 0 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   VAR7 >= 0.15: 0.5 (1/0)
|   |   |   |   |   |   |   |   |   |   |   VAR1 >= 0.25: 0 (7/0)
|   |   |   |   |   |   |   |   |   VAR8 >= 5.5: 0 (7/0)
|   |   |   |   |   |   |   VAR7 >= 0.25
|   |   |   |   |   |   |   |   VAR3 < 0.45: 0 (56/0)
|   |   |   |   |   |   |   |   VAR3 >= 0.45
|   |   |   |   |   |   |   |   |   VAR7 < 0.75
|   |   |   |   |   |   |   |   |   |   VAR9 < 1.5: 0 (37/0)
|   |   |   |   |   |   |   |   |   |   VAR9 >= 1.5
|   |   |   |   |   |   |   |   |   |   |   VAR3 < 0.75
|   |   |   |   |   |   |   |   |   |   |   |   VAR5 < 0.75
|   |   |   |   |   |   |   |   |   |   |   |   |   VAR1 < 0.65
|   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR8 < 4.5: 0.1 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR8 >= 4.5
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR1 < 0.55
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR5 < 0.3: 0.1 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR5 >= 0.3
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR8 < 5.5
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR3 < 0.55: 0.05 (2/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR3 >= 0.55: 0 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR8 >= 5.5: 0 (4/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR1 >= 0.55: 0 (3/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   VAR1 >= 0.65: 0 (4/0)
|   |   |   |   |   |   |   |   |   |   |   |   VAR5 >= 0.75: 0.1 (1/0)
|   |   |   |   |   |   |   |   |   |   |   VAR3 >= 0.75: 0 (3/0)
|   |   |   |   |   |   |   |   |   VAR7 >= 0.75: 0 (17/0)
|   |   |   |   |   |   VAR5 >= 0.85
```

```
|   |   |   |   |   |   |   VAR3 < 0.95: 0 (6/0)
|   |   |   |   |   |   |   VAR3 >= 0.95
|   |   |   |   |   |   |   |   VAR1 < 0.95
|   |   |   |   |   |   |   |   |   VAR7 < 0.7: 0.2 (1/0)
|   |   |   |   |   |   |   |   |   VAR7 >= 0.7: 0 (1/0)
|   |   |   |   |   |   |   |   VAR1 >= 0.95
|   |   |   |   |   |   |   |   |   VAR5 < 0.95: 0 (1/0)
|   |   |   |   |   |   |   |   |   VAR5 >= 0.95
|   |   |   |   |   |   |   |   |   |   VAR9 < 1.5: 0 (2/0)
|   |   |   |   |   |   |   |   |   |   VAR9 >= 1.5: 0.05 (2/0)
|   |   |   |   |   VAR9 >= 2.5
|   |   |   |   |   |   VAR7 < 0.35
|   |   |   |   |   |   |   VAR8 < 2.5
|   |   |   |   |   |   |   |   VAR5 < 0.5
|   |   |   |   |   |   |   |   |   VAR9 < 3.5
|   |   |   |   |   |   |   |   |   |   VAR5 < 0.05
|   |   |   |   |   |   |   |   |   |   |   VAR8 < 1.5: 0 (1/0)
|   |   |   |   |   |   |   |   |   |   |   VAR8 >= 1.5: 0.1 (1/0)
|   |   |   |   |   |   |   |   |   |   VAR5 >= 0.05: 0.2 (1/0)
|   |   |   |   |   |   |   |   |   VAR9 >= 3.5
|   |   |   |   |   |   |   |   |   |   VAR5 < 0.1
|   |   |   |   |   |   |   |   |   |   |   VAR9 < 4.5: 0.05 (2/0)
|   |   |   |   |   |   |   |   |   |   |   VAR9 >= 4.5: 0 (1/0)
|   |   |   |   |   |   |   |   |   |   VAR5 >= 0.1: 0 (2/0)
|   |   |   |   |   |   |   |   VAR5 >= 0.5: 0.4 (1/0)
|   |   |   |   |   |   |   VAR8 >= 2.5
|   |   |   |   |   |   |   |   VAR1 < 0.35
|   |   |   |   |   |   |   |   |   VAR9 < 3.5
|   |   |   |   |   |   |   |   |   |   VAR1 < 0.15: 0 (9/0)
|   |   |   |   |   |   |   |   |   |   VAR1 >= 0.15
|   |   |   |   |   |   |   |   |   |   |   VAR3 < 0.4
|   |   |   |   |   |   |   |   |   |   |   |   VAR5 < 0.15
|   |   |   |   |   |   |   |   |   |   |   |   |   VAR1 < 0.25
|   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR3 < 0.15: 0 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR3 >= 0.15: 0.1 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   VAR1 >= 0.25: 0 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   VAR5 >= 0.15: 0 (5/0)
|   |   |   |   |   |   |   |   |   |   |   VAR3 >= 0.4: 0.1 (1/0)
|   |   |   |   |   |   |   |   |   VAR9 >= 3.5
|   |   |   |   |   |   |   |   |   |   VAR3 < 0.45
|   |   |   |   |   |   |   |   |   |   |   VAR3 < 0.25
|   |   |   |   |   |   |   |   |   |   |   |   VAR5 < 0.35
|   |   |   |   |   |   |   |   |   |   |   |   |   VAR8 < 6.5
|   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR1 < 0.25
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR3 < 0.05
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR7 < 0.05
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR9 < 4.5: 0.15 (2/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR9 >= 4.5: 0 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR7 >= 0.05: 0 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR3 >= 0.05
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR5 < 0.05: 0 (4/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR5 >= 0.05
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR7 < 0.25
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR8 < 4.5
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR9 < 4.5
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR8 < 3.5: 0 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR8 >= 3.5: 0.05 (2/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR9 >= 4.5: 0 (4/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR8 >= 4.5
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR8 < 5.5
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR9 < 4.5
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR5 < 0.15
```

```
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR1 < 0.15: 0.13
(3/0.02)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR1 >= 0.15: 0
(1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR5 >= 0.15: 0 (3/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR9 >= 4.5: 0.3 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR8 >= 5.5
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR3 < 0.15
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR1 < 0.15: 0 (2/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR1 >= 0.15: 0.1 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR3 >= 0.15: 0 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR7 >= 0.25: -0 (2/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR1 >= 0.25: 0.1 (2/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   VAR8 >= 6.5: 0 (4/0)
|   |   |   |   |   |   |   |   |   |   |   |   VAR5 >= 0.35: 0 (2/0)
|   |   |   |   |   |   |   |   |   |   |   VAR3 >= 0.25
|   |   |   |   |   |   |   |   |   |   |   |   VAR7 < 0.15: 0 (2/0)
|   |   |   |   |   |   |   |   |   |   |   |   VAR7 >= 0.15
|   |   |   |   |   |   |   |   |   |   |   |   |   VAR7 < 0.25
|   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR3 < 0.35
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR8 < 6.5: 0 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR8 >= 6.5: 0.1 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR3 >= 0.35: 0 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   VAR7 >= 0.25: 0 (2/0)
|   |   |   |   |   |   |   |   |   |   VAR3 >= 0.45
|   |   |   |   |   |   |   |   |   |   |   VAR3 < 0.75: 0.2 (1/0)
|   |   |   |   |   |   |   |   |   |   |   VAR3 >= 0.75: 0 (1/0)
|   |   |   |   |   |   |   |   VAR1 >= 0.35
|   |   |   |   |   |   |   |   |   VAR7 < 0.15: 0 (6/0)
|   |   |   |   |   |   |   |   |   VAR7 >= 0.15
|   |   |   |   |   |   |   |   |   |   VAR9 < 3.5: 0 (6/0)
|   |   |   |   |   |   |   |   |   |   VAR9 >= 3.5
|   |   |   |   |   |   |   |   |   |   |   VAR3 < 0.45
|   |   |   |   |   |   |   |   |   |   |   |   VAR1 < 0.6: 0 (4/0)
|   |   |   |   |   |   |   |   |   |   |   |   VAR1 >= 0.6
|   |   |   |   |   |   |   |   |   |   |   |   |   VAR1 < 0.85: 0.1 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   VAR1 >= 0.85: 0 (1/0)
|   |   |   |   |   |   |   |   |   |   |   VAR3 >= 0.45
|   |   |   |   |   |   |   |   |   |   |   |   VAR9 < 4.5
|   |   |   |   |   |   |   |   |   |   |   |   |   VAR7 < 0.25: 0.1 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   VAR7 >= 0.25
|   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR8 < 5.5: 0.1 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   |   |   VAR8 >= 5.5: 0 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   VAR9 >= 4.5: 0 (1/0)
|   |   |   |   |   |   VAR7 >= 0.35
|   |   |   |   |   |   |   VAR1 < 0.25
|   |   |   |   |   |   |   |   VAR9 < 4.5
|   |   |   |   |   |   |   |   |   VAR5 < 0.15: 0 (5/0)
|   |   |   |   |   |   |   |   |   VAR5 >= 0.15
|   |   |   |   |   |   |   |   |   |   VAR3 < 0.35: 0.3 (1/0)
|   |   |   |   |   |   |   |   |   |   VAR3 >= 0.35: 0 (2/0)
|   |   |   |   |   |   |   |   VAR9 >= 4.5
|   |   |   |   |   |   |   |   |   VAR3 < 0.35
|   |   |   |   |   |   |   |   |   |   VAR3 < 0.05: 0.1 (1/0)
|   |   |   |   |   |   |   |   |   |   VAR3 >= 0.05
|   |   |   |   |   |   |   |   |   |   |   VAR5 < 0.15
|   |   |   |   |   |   |   |   |   |   |   |   VAR3 < 0.15: 0.4 (2/0)
|   |   |   |   |   |   |   |   |   |   |   |   VAR3 >= 0.15: 0.3 (1/0)
|   |   |   |   |   |   |   |   |   |   |   VAR5 >= 0.15
|   |   |   |   |   |   |   |   |   |   |   |   VAR7 < 0.75: 0.5 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   VAR7 >= 0.75: 0 (1/0)
|   |   |   |   |   |   |   |   |   VAR3 >= 0.35: -0 (2/0)
|   |   |   |   |   |   |   VAR1 >= 0.25
```

```
|   |   |   |   |   |   |   |   VAR8 < 5.5
|   |   |   |   |   |   |   |   |   VAR3 < 0.45: 0 (9/0)
|   |   |   |   |   |   |   |   |   VAR3 >= 0.45
|   |   |   |   |   |   |   |   |   |   VAR7 < 0.55
|   |   |   |   |   |   |   |   |   |   |   VAR9 < 3.5
|   |   |   |   |   |   |   |   |   |   |   |   VAR7 < 0.45: 0 (1/0)
|   |   |   |   |   |   |   |   |   |   |   |   VAR7 >= 0.45: 0.1 (1/0)
|   |   |   |   |   |   |   |   |   |   |   VAR9 >= 3.5
```

```
Size of the tree: 17797

=== Cross-validation ===
Correlation coefficient                  0.7822
Mean absolute error                      0.1128
Root mean squared error                  0.1932
Relative absolute error                 45.691%
Root relative squared error             65.821%
Total Number of Instances                16,142
```

Factor Analysis with Eigenvalue and Eigenvector Analysis

```
Correlation Matrix
                    VAR1      VAR2      VAR3      VAR4      VAR5      VAR6      VAR7      VAR8      VAR9     VAR10     VAR11     VAR12     VAR13
VAR1              1.0000    0.5838    0.6181    0.6223    0.4292    0.4300    0.8203    0.8525    0.6274    0.3739    0.4231    0.5712    0.5571
VAR2              0.5838    1.0000    0.5371    0.7586    0.6079    0.7490    0.5476    0.6184    0.7322    0.6276    0.6918    0.4887    0.7941
VAR3              0.6181    0.5371    1.0000    0.5697    0.3969    0.4879    0.6268    0.7091    0.5606    0.4130    0.3905    0.6656    0.5197
VAR4              0.6223    0.7586    0.5697    1.0000    0.5445    0.7219    0.5416    0.6199    0.9232    0.5364    0.6784    0.5037    0.7111
VAR5              0.4292    0.6079    0.3969    0.5445    1.0000    0.6533    0.3171    0.4521    0.4943    0.7835    0.5014    0.3119    0.6263
VAR6              0.4300    0.7490    0.4879    0.7219    0.6533    1.0000    0.3716    0.4894    0.6496    0.6706    0.6065    0.3909    0.8073
VAR7              0.8203    0.5476    0.6268    0.5416    0.3171    0.3716    1.0000    0.8369    0.5739    0.3084    0.4235    0.6001    0.4993
VAR8              0.8525    0.6184    0.7091    0.6199    0.4521    0.4894    0.8369    1.0000    0.6307    0.4034    0.4453    0.6067    0.5961
VAR9              0.6274    0.7322    0.5606    0.9232    0.4943    0.6496    0.5739    0.6307    1.0000    0.5422    0.6991    0.4887    0.7331
VAR10             0.3739    0.6276    0.4130    0.5364    0.7835    0.6706    0.3084    0.4034    0.5422    1.0000    0.6097    0.2861    0.6921
VAR11             0.4231    0.6918    0.3905    0.6784    0.5014    0.6065    0.4235    0.4453    0.6991    0.6097    1.0000    0.2788    0.6650
VAR12             0.5712    0.4887    0.6656    0.5037    0.3119    0.3909    0.6001    0.6067    0.4887    0.2861    0.2788    1.0000    0.3902
VAR13             0.5571    0.7941    0.5197    0.7111    0.6263    0.8073    0.4993    0.5961    0.7331    0.6921    0.6650    0.3902    1.0000


Eigenvalues and Eigenvectors

Eigenvalue        7.9266    1.7063    0.7530    0.5981    0.4007    0.3675    0.3355    0.2211    0.2004    0.1617    0.1478    0.1262    0.0551
Proportion        0.6097    0.1313    0.0579    0.0460    0.0042    0.0308    0.0283    0.0258    0.0170    0.0154    0.0097    0.0114    0.0124
Cum Proportion    0.6097    0.7410    0.7989    0.8449    0.8492    0.8800    0.9083    0.9341    0.9511    0.9665    0.9762    0.9876    1.0000


Eigenvectors    Factor 1  Factor 2  Factor 3  Factor 4  Factor 5  Factor 6  Factor 7  Factor 8  Factor 9 Factor 10 Factor 11 Factor 12 Factor 13
VAR1             -0.2771    0.3350   -0.0167    0.3903   -0.1115    0.1998    0.0179    0.0218   -0.0069   -0.4040   -0.5578   -0.3526   -0.0805
VAR2             -0.3087   -0.1293    0.1045   -0.0464   -0.1652   -0.1990    0.3700   -0.4517   -0.6324    0.2039   -0.1404    0.0121   -0.0757
VAR3             -0.2608    0.2704   -0.2324   -0.3646    0.1669   -0.3369   -0.6604   -0.1947   -0.1279   -0.0084   -0.0024   -0.1991   -0.0217
VAR4             -0.3093   -0.0631    0.3601   -0.1848   -0.0523    0.4456   -0.1785   -0.0739    0.0431    0.1557   -0.0724   -0.0037    0.6830
VAR5             -0.2482   -0.3004   -0.5241    0.1748    0.0750    0.4231    0.0101   -0.3977    0.1023   -0.1159    0.3874   -0.1468   -0.0670
VAR6             -0.2835   -0.2788   -0.0490   -0.2212   -0.4906   -0.1792    0.0143   -0.0588    0.5910    0.2052   -0.2692   -0.0036   -0.2240
VAR7             -0.2608    0.4034    0.0243    0.3428    0.0535   -0.1468    0.1725    0.1336    0.1660    0.5853    0.3260   -0.3168    0.0475
VAR8             -0.2893    0.3297   -0.0854    0.2762   -0.1163   -0.0566   -0.1017   -0.1099    0.0735   -0.0866    0.0742    0.8155    0.0361
VAR9             -0.3067   -0.0400    0.4169   -0.1059    0.0657    0.3760   -0.2024    0.2493   -0.1459    0.0481    0.1875    0.0602   -0.6388
VAR10            -0.2532   -0.3617   -0.3983    0.1218    0.3163   -0.0131   -0.0537    0.5114   -0.1924    0.2758   -0.3510    0.1548    0.0818
VAR11            -0.2616   -0.2457    0.3697    0.1252    0.6249   -0.3380    0.1317   -0.1940    0.3118   -0.2473   -0.0207    0.0057    0.0141
VAR12            -0.2270    0.3472   -0.2229   -0.6010    0.1897    0.1285    0.5413    0.1717    0.0890   -0.1733    0.0266    0.0566    0.0119
VAR13            -0.3039   -0.2028    0.0356    0.0452   -0.3689   -0.3167    0.0148    0.4120   -0.1600   -0.4424    0.4125   -0.1344    0.2178


Unrotated Factor Loadings
                Factor 1  Factor 2  Factor 3  Factor 4  Factor 5  Factor 6  Factor 7  Factor 8  Factor 9 Factor 10 Factor 11 Factor 12 Factor 13
VAR1             -0.7803    0.4376   -0.0145    0.3019   -0.0706    0.1211    0.0104    0.0102   -0.0031   -0.1624   -0.2145   -0.1252   -0.0189
VAR2             -0.8691   -0.1689    0.0907   -0.0359   -0.1046   -0.1206    0.2143   -0.2124   -0.2831    0.0820   -0.0540    0.0043   -0.0178
VAR3             -0.7343    0.3532   -0.2017   -0.2820    0.1056   -0.2042   -0.3825   -0.0915   -0.0572   -0.0034   -0.0009   -0.0707   -0.0051
VAR4             -0.8709   -0.0824    0.3125   -0.1429   -0.0331    0.2701   -0.1034   -0.0347    0.0193    0.0626   -0.0278   -0.0013    0.1603
VAR5             -0.6989   -0.3924   -0.4548    0.1352    0.0475    0.2565    0.0058   -0.1870    0.0458   -0.0466    0.1489   -0.0522   -0.0157
VAR6             -0.7980   -0.3642   -0.0425   -0.1711   -0.3106   -0.1086    0.0083   -0.0276    0.2646    0.0825   -0.1035   -0.0013   -0.0526
VAR7             -0.7342    0.5270    0.0211    0.2652    0.0339   -0.0890    0.0999    0.0628    0.0743    0.2354    0.1253   -0.1125    0.0111
VAR8             -0.8146    0.4307   -0.0741    0.2136   -0.0736   -0.0343   -0.0589   -0.0517    0.0329   -0.0348    0.0285    0.2897    0.0085
VAR9             -0.8636   -0.0523    0.3618   -0.0819    0.0416    0.2279   -0.1172    0.1172   -0.0653    0.0193    0.0721    0.0214   -0.1499
VAR10            -0.7128   -0.4725   -0.3456    0.0942    0.2002   -0.0079   -0.0311    0.2404   -0.0861    0.1109   -0.1349    0.0550    0.0192
VAR11            -0.7366   -0.3209    0.3208    0.0969    0.3956   -0.2049    0.0763   -0.0912    0.1396   -0.0995   -0.0080    0.0020    0.0033
VAR12            -0.6392    0.4535   -0.1934   -0.4648    0.1201    0.0779    0.3135    0.0807    0.0398   -0.0697    0.0102    0.0201    0.0028
VAR13            -0.8555   -0.2649    0.0309    0.0349   -0.2335   -0.1920    0.0086    0.1937   -0.0716   -0.1779    0.1586   -0.0477    0.0511
```

```
Varimax Rotated Factor Loadings
                Factor 1  Factor 2  Factor 3  Factor 4  Factor 5  Factor 6  Factor 7  Factor 8  Factor 9 Factor 10 Factor 11 Factor 12 Factor 13
VAR1             -0.1692    0.8392    0.0900   -0.1748   -0.0837    0.2614   -0.1362    0.0911   -0.0985    0.0534   -0.3337   -0.0481    0.0066
VAR2             -0.2988    0.3006    0.2935   -0.1762   -0.2827    0.3610   -0.1276    0.1862   -0.6539    0.1182   -0.0049    0.0191   -0.0017
VAR3             -0.1454    0.4022    0.0975   -0.3013   -0.1340    0.2049   -0.7985    0.0804   -0.0811    0.0750   -0.0084    0.0227   -0.0022
VAR4             -0.2287    0.2962    0.2347   -0.1709   -0.2561    0.7852   -0.1544    0.0815   -0.1591    0.0615   -0.0288    0.0193    0.1809
VAR5             -0.9182    0.1719    0.1340   -0.0840   -0.1759    0.1884   -0.0884    0.0882   -0.1092    0.0889   -0.0120    0.0168    0.0023
VAR6             -0.3657    0.1384    0.2071   -0.1285   -0.7611    0.3291   -0.1401    0.1833   -0.1730    0.1283   -0.0068    0.0187    0.0043
VAR7             -0.0548    0.8818    0.1509   -0.2221   -0.0777    0.1733   -0.1555    0.0663   -0.0968    0.0526    0.2356   -0.1224   -0.0022
VAR8             -0.1780    0.7957    0.1082   -0.1978   -0.1390    0.2206   -0.2496    0.1074   -0.1159    0.0508   -0.0007    0.3556   -0.0019
VAR9             -0.1750    0.3282    0.2751   -0.1545   -0.1516    0.7956   -0.1423    0.1692   -0.1215    0.1135    0.0022    0.0215   -0.1709
VAR10            -0.6059    0.1098    0.2611   -0.0683   -0.2087    0.1924   -0.1213    0.1602   -0.1147    0.6433    0.0014    0.0042   -0.0099
VAR11            -0.2339    0.1829    0.8418   -0.0507   -0.1699    0.3412   -0.0782    0.1174   -0.1426    0.1227   -0.0015    0.0090    0.0055
VAR12            -0.0935    0.3548    0.0370   -0.8727   -0.0900    0.1791   -0.2254    0.0473   -0.0875    0.0359   -0.0078    0.0165    0.0021
VAR13            -0.3285    0.2824    0.2596   -0.0803   -0.3664    0.3414   -0.1366    0.6296   -0.2134    0.1625   -0.0081    0.0226   -0.0071


Sum of Squares    1.7719    2.8624    1.1754    1.0927    1.0439    2.0148    0.9207    0.5802    0.6353    0.5253    0.1681    0.1470    0.0622
Rank                   3         1         4         5         6         2         7         9         8        10        11        12        13

Varimax Rotated Factor Loadings (Ranked)
                Factor 1  Factor 2  Factor 3  Factor 4  Factor 5  Factor 6  Factor 7  Factor 8  Factor 9 Factor 10 Factor 11 Factor 12 Factor 13
VAR1              0.8392    0.2614   -0.1692    0.0900   -0.1748   -0.0837   -0.1362   -0.0985    0.0911    0.0534   -0.3337   -0.0481    0.0066
VAR2              0.3006    0.3610   -0.2988    0.2935   -0.1762   -0.2827   -0.1276   -0.6539    0.1862    0.1182   -0.0049    0.0191   -0.0017
VAR3              0.4022    0.2049   -0.1454    0.0975   -0.3013   -0.1340   -0.7985   -0.0811    0.0804    0.0750   -0.0084    0.0227   -0.0022
VAR4              0.2962    0.7852   -0.2287    0.2347   -0.1709   -0.2561   -0.1544   -0.1591    0.0815    0.0615   -0.0288    0.0193    0.1809
VAR5              0.1719    0.1884   -0.9182    0.1340   -0.0840   -0.1759   -0.0884   -0.1092    0.0882    0.0889   -0.0120    0.0168    0.0023
VAR6              0.1384    0.3291   -0.3657    0.2071   -0.1285   -0.7611   -0.1401   -0.1730    0.1833    0.1283   -0.0068    0.0187    0.0043
VAR7              0.8818    0.1733   -0.0548    0.1509   -0.2221   -0.0777   -0.1555   -0.0968    0.0663    0.0526    0.2356   -0.1224   -0.0022
VAR8              0.7957    0.2206   -0.1780    0.1082   -0.1978   -0.1390   -0.2496   -0.1159    0.1074    0.0508   -0.0007    0.3556   -0.0019
VAR9              0.3282    0.7956   -0.1750    0.2751   -0.1545   -0.1516   -0.1423   -0.1215    0.1692    0.1135    0.0022    0.0215   -0.1709
VAR10             0.1098    0.1924   -0.6059    0.2611   -0.0683   -0.2087   -0.1213   -0.1147    0.1602    0.6433    0.0014    0.0042   -0.0099
VAR11             0.1829    0.3412   -0.2339    0.8418   -0.0507   -0.1699   -0.0782   -0.1426    0.1174    0.1227   -0.0015    0.0090    0.0055
VAR12             0.3548    0.1791   -0.0935    0.0370   -0.8727   -0.0900   -0.2254   -0.0875    0.0473    0.0359   -0.0078    0.0165    0.0021
VAR13             0.2824    0.3414   -0.3285    0.2596   -0.0803   -0.3664   -0.1366   -0.2134    0.6296    0.1625   -0.0081    0.0226   -0.0071


Sum of Squares    2.8624    2.0148    1.7719    1.1754    1.0927    1.0439    0.9207    0.6353    0.5802    0.5253    0.1681    0.1470    0.0622
Rank                   1         2         3         4         5         6         7         8         9        10        11        12        13
Proportion        22.02%    15.50%    13.63%     9.04%     8.41%     8.03%     7.08%     4.89%     4.46%     4.04%     1.29%     1.13%     0.48%
Cum Proportion    22.02%    37.52%    51.15%    60.19%    68.59%    76.62%    83.71%    88.59%    93.06%    97.10%    98.39%    99.52%   100.00%


Factor Score Coefficients
                Factor 1  Factor 2  Factor 3  Factor 4  Factor 5  Factor 6  Factor 7  Factor 8  Factor 9 Factor 10 Factor 11 Factor 12 Factor 13
VAR1              0.4956   -0.1330    0.0505   -0.0109    0.1279   -0.0778    0.1564    0.0688   -0.1067    0.0710   -1.9190   -0.8076   -0.1470
VAR2             -0.0996   -0.1175    0.0912   -0.1436    0.0786    0.1988   -0.0130   -1.9429   -0.3212   -0.0031   -0.0268   -0.0400   -0.2439
VAR3             -0.1884   -0.0969    0.0264    0.0115    0.2702    0.1109   -1.5202   -0.0174   -0.0459   -0.1307   -0.0874   -0.3555   -0.0642
VAR4             -0.1040    0.8625    0.0568   -0.2555    0.0757    0.2146    0.0386    0.1528   -0.0810    0.0941    0.2086    0.0336    2.9350
VAR5             -0.0265   -0.0400   -1.3162   -0.0788    0.0097    0.3454    0.0164    0.1593   -0.0891   -1.0277    0.2344   -0.1739   -0.2472
VAR6              0.0270   -0.2013    0.1671   -0.0557    0.0299   -1.7733    0.1003    0.3298   -0.6653   -0.1024   -0.0925   -0.1219   -0.6611
VAR7              0.6717   -0.1154    0.0203   -0.0426    0.1558   -0.0984    0.1881    0.1071   -0.1156    0.0310    1.5553   -1.1587    0.1575
VAR8              0.3438   -0.1125    0.0497   -0.0101    0.1070   -0.0332    0.1518    0.0738   -0.1117    0.0811    0.2639    2.3404    0.0646
VAR9             -0.1000    0.9017    0.0419   -0.2106    0.0598    0.1657    0.0647    0.1897   -0.1776   -0.0207    0.1142   -0.0031   -2.8055
VAR10            -0.0127   -0.0359    0.0865   -0.1535   -0.0018    0.1292    0.0732    0.0650   -0.2277    1.8387    0.0010    0.0703    0.2566
VAR11            -0.0638   -0.3263    0.0498    1.4811   -0.0950    0.0805    0.0037    0.3232   -0.1536   -0.3318   -0.2163    0.0997    0.1110
VAR12            -0.2144   -0.1242    0.0165    0.0764   -1.3673    0.0842    0.3699    0.1649    0.1465    0.0029   -0.0997    0.0827    0.0086
VAR13            -0.0802   -0.1127    0.0364   -0.0771   -0.0259    0.2957    0.0287    0.2647    2.0757   -0.2488    0.0718   -0.0892    0.5883
```